Public Goods

$U^1(x_1, y_1)$
$U^2(x_2, y_2)$
$g(L_x, K_x)$
$f(L_y, K_y)$
$\bar{K}, \bar{L}$

feasibility....
$x_1 + x_2 \leq g(L_x, K_x)$

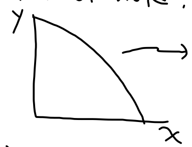
x is a "public good."
Nonrival in consumption.
Real resources required to produce it, but once it's produced, any number of people can consume it.
"services"

$U^1(x, y_1), U^2(x, y_2)$
$x \leq g(L_x, K_x)$

Allocation

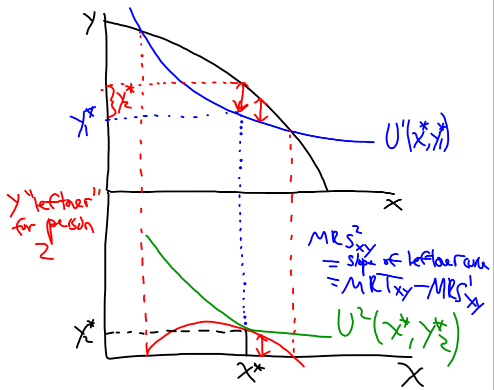
$(x^*, y_1^*, y_2^*, L_x^*, L_y^*, K_x^*, K_y^*)$
↑
total

If this is an efficient allocation,
Then what must hold?



→ Still must be on PPF, equalize $MRTS_{KL}$.

Other two conditions change.
One condition instead, called "Samuelson Condition."



$$\Longrightarrow \boxed{MRS^1_{xy} + MRS^2_{xy} = MRT_{xy}}$$

"Samuelson Condition"

Alternatively,

$$\max_{x, y_1, y_2} \; U^2(x, y_2)$$

subject to $U^1(x, y_1) = U^1(x^*, y_1^*)$

$\boxed{PPF} \leftarrow h(x, y_1 + y_2) = 0$

$$\Longrightarrow \frac{\partial U^2/\partial x}{\partial U^2/\partial y_2} + \frac{\partial U^1/\partial x}{\partial U^1/\partial y_1} = \frac{\partial h/\partial x}{\partial h/\partial y}$$

Slope of PPF

Suppose it didn't hold.
Why is the allocation inefficient?

$$2 + 8 > 5$$

$$MRS^1_{xy} + MRS^2_{xy} > MRT_{xy}$$

"Total willingness to pay for public good."

"opportunity cost of public good"

Amt. of "y" people willing to forego in total exceeds the cost of an extra unit of public good.

So, for example, take:

$1y$ from person 1

$4y$ from person 2

$\Longrightarrow 5y$ total; convert to $1x$, both are better off.

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Important: These people need not have the same preferences! They just both benefit from this reallocation.

② Could have done the reallocation in a way that led to an efficient allocation but made someone worse off. Take 5 away from person 1 and nothing from person 2, say. Produce the extra "X" by taking from person 1.

Assume a given distribution of income.

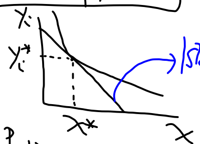
Derive demand curves for the public good, one for each person.

$$\text{Max } U^i(X, Y_i)$$

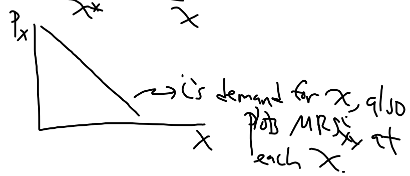
$$\text{s.t. } P_x \cdot X + P_y \cdot Y_i = I^i$$

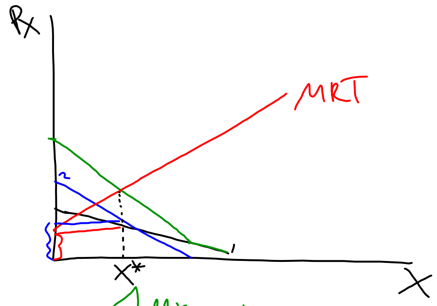
$\boxed{\text{Suppose } P_y = 1}$ (normalize prices).

$\Longrightarrow$



$|\text{slope}| = \frac{P_x}{P_y} = \boxed{P_x = MRS^i_{xy}}$





— = $\sum$ MRS at each X

Equilibrium question: what will "actually" happen? With what institutions?

→ Voluntary contribution, one possibility.

General result is $\boxed{\text{underprovision}}$, so not X^* above.

More generally, an inefficient allocation.

Model this as a prisoner's dilemma, which is a special case of the voluntary contribution game.

(A) \ (B)	Give	Don't Give
Give	9, 9	0, 10
Don't Give	10, 0	2, 2

Strong prediction: both don't give.

Both would be better off if both give. So, both should agree to coercion. Not enough to have coordination.

Gruber, "Business Improvement Districts"

This is not a comparison between voluntary contribution and government, it is a comparison between two forms of government.

One govt (NYC) could not in itself impose targeted tax, ~~so~~ and the alternative city-wide tax to improve Times Square was unacceptable to those outside Manhattan. So, stuck.

[Coasian bargain to share revenue from revitalized Times Square infeasible?]

Solution was a new, "single-function government," the BID.