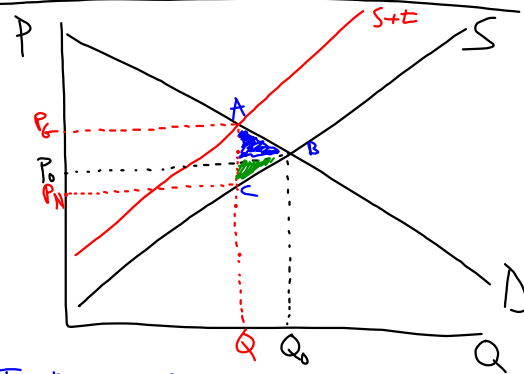


Total Burden of Taxation



$$\text{Tax Revenue} = P_G - P_N \cdot Q = t \cdot Q$$

$$\text{Total Burden} = P_G - A - B - C - P_N$$

Consumer's Part, $P_G - A - B - P_0$ = EB

Producer's Part, $P_0 - B - C - P_N$ = lost profit

$$\text{Excess Burden} = ABC$$

Magnitude of Total Burden,
Tax Revenue + Excess Burden

Sharing of Total Burden,
Depends on elasticities.

Conclusion: Should not focus on just tax
revenue raised in discussing "tax burden"

Should not focus on just
prices or incomes in discussing "who pays
the tax." How is total burden shared
is the real incidence question.

Optimal Commodity Tax Problem

Want to use a system of commodity taxes to raise a specified amount of revenue at the lowest DWL.

Overview:

Goods x, y so 2 possible tax bases.

$F(t_x, t_y)$: Total Excess Burden

$G(t_x, t_y)$: Revenue from the taxes.

Min $F(t_x, t_y)$

t_x, t_y s.t. $G(t_x, t_y) = R$

$$\mathcal{L} = F(t_x, t_y) + \lambda [G(t_x, t_y) - R]$$

$$\frac{\partial \mathcal{L}}{\partial t_x} = \frac{\partial F}{\partial t_x} + \lambda \frac{\partial G}{\partial t_x} = 0$$

$$\frac{\partial \mathcal{L}}{\partial t_y} = \frac{\partial F}{\partial t_y} + \lambda \frac{\partial G}{\partial t_y} = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = G(t_x, t_y) - R = 0$$

$$\Rightarrow \frac{\partial F / \partial t_x}{\partial F / \partial t_y} = \frac{\partial G / \partial t_x}{\partial G / \partial t_y}$$

$$\Rightarrow \frac{\partial F / \partial t_x}{\partial G / \partial t_x} = \frac{\partial F / \partial t_y}{\partial G / \partial t_y}$$

$$\Rightarrow \frac{\partial F / \partial t_x}{\partial G / \partial t_x} = \frac{\partial F / \partial t_y}{\partial G / \partial t_y}$$

Ratio of extra burden to extra tax revenue from increasing t_x = Ratio of extra burden to extra tax revenue from increasing t_y

General, but not too helpful.

With lots of assumptions, can get a more enlightening conclusion, "inverse elasticity rule".

$F(t_x, t_y)$:

$$\Rightarrow EB_x(t_x, t_y) + EB_y(t_x, t_y)$$

$$\Rightarrow EB_x(t_x) + EB_y(t_y), \text{ st. plot.}$$

$$\Rightarrow \frac{1}{2} t_x^2 \cdot \frac{\partial X_D}{\partial P_x} + \frac{1}{2} t_y^2 \cdot \frac{\partial Y_D}{\partial P_y}$$

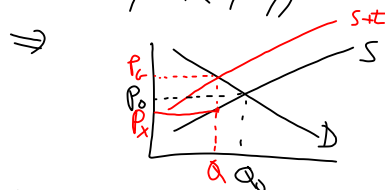
$G(t_x, t_y)$:

$$\Rightarrow t_x \cdot X(P_x + t_x, P_y + t_y) + t_y \cdot Y(P_x + t_x, P_y + t_y)$$

All are gross prices.

$$\Rightarrow t_x \cdot X^D(P_x + t_x) + t_y \cdot Y^D(P_y + t_y)$$

$P_x = \text{net}$,
 $P_x + t = \text{gross}$



Assume supply is horizontal. Then $P_x = P_0$

$$G(t_x, t_y) = t_x \cdot X^D(P_0 + t_x) + t_y \cdot Y^D(P_0 + t_y)$$

$$G(t_x, t_y) = t_x \cdot X^D(P_0^X + t_x) + t_y \cdot Y^D(P_0^Y + t_y)$$

Assume $P_0^X = P_0^Y = 1$, without any loss of generality.

$$G(t_x, t_y) = t_x \cdot X^D(1+t_x) + t_y \cdot Y^D(1+t_y)$$

$$\frac{\partial F}{\partial t_x} = \frac{\partial}{\partial t_x} \left(\frac{1}{2} t_x^2 \frac{\partial X_D}{\partial P_x} \right)$$

$\frac{\partial X_D}{\partial P_x}$ constant

$$= t_x \cdot \frac{\partial X_D}{\partial P_x} + 0$$

$$\frac{\partial F}{\partial t_y} = t_y \cdot \frac{\partial Y_D}{\partial P_y}$$

$$\frac{\partial G}{\partial t_x} = \frac{\partial}{\partial t_x} (t_x \cdot X_D(1+t_x))$$

$$= X_D + t_x \cdot \frac{\partial X_D}{\partial P_x}$$

$$\frac{\partial G}{\partial t_y} = Y_D + t_y \cdot \frac{\partial Y_D}{\partial P_y}$$

Form ratios:

constant by assumption.

$$t_x \cdot \frac{\partial X_D}{\partial P_x} = t_y \cdot \frac{\partial Y_D}{\partial P_y}$$

$$\therefore X_D + t_x \cdot \frac{\partial X_D}{\partial P_x} = Y_D + t_y \cdot \frac{\partial Y_D}{\partial P_y}$$

$P_x = P_0$, X_D is original (pre-tax) quantity, Y_D is " " " "

$$t_x \cdot \left(\frac{\partial X_D}{\partial R} \right) = t_y \cdot \left(\frac{\partial Y_D}{\partial R} \right)$$

constituted by description.

$$X_D + t_x \cdot \frac{\partial X_D}{\partial R} \quad Y_D + t_y \cdot \frac{\partial Y_D}{\partial R}$$

$$t_x \cdot \left(\frac{P_x}{X_D} \right) \cdot \frac{\partial X_D}{\partial R} \Rightarrow t_x \cdot E_D^x$$

$$\cancel{X_D \left(\frac{R}{X_D} \right)} + t_x \cdot \left(\frac{P_x}{X_D} \right) \cdot \frac{\partial X_D}{\partial R} \quad | + t_x \cdot E_D^x$$

(Using $P_x = P_0 = 1$)

Do same for Y ,

$$\Rightarrow \frac{t_x \cdot E_D^x}{1 + t_x \cdot E_D^x} = \frac{t_y \cdot E_D^y}{1 + t_y \cdot E_D^y}$$

$$\Rightarrow t_x \cdot E_D^x = t_y \cdot E_D^y$$

$$\Rightarrow \left| E_D^x \right| > \left| E_D^y \right| \Rightarrow t_x < t_y$$

Inverse elasticity rule.

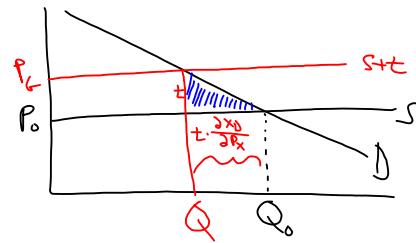
Flatter (more) demand curves,

Smaller optimal tax rates.

* Efficient taxes will tax more price responsive goods and services less.

If one group has inelastic demands and another has elastic demands, former will pay most of the tax. Fair?

* Recall, EB increases with the square of the tax rate.



$$EB_x = \frac{1}{2} t^2 \cdot \frac{2X_0}{\frac{\partial X}{\partial P_x}}$$

So extra excess burden depends on the level of initial taxes.

Need a little extra revenue, seriously consider finding a new base or broadening existing bases rather than run up existing tax rates higher.

More generally adding a tax to any market that has a distortion in it (not necessarily a tax distortion) is likely to cause large additional burden.

Be careful! For example, if there is a negative externality, then the tax actually helps.

Page 76, course reader.

For US economy, CGE model.

$$MEC: \frac{\text{extra excess burden}}{\text{extra revenue}}$$

$$AEC: \frac{\text{total EB}}{\text{total revenue}}$$

$$AEC = .238$$

On every dollar of revenue, there is an additional 24 cents of burden that is lost (or wasted).

$$MEC = .332$$

On an extra dollar of revenue, raised by increasing all tax rates, there is additional 33¢ of excess burden.

MEC across different taxes:

$$\text{Sales taxes } (.256) < \text{labor income } (.482) < \text{capital income } (.838)$$

All these numbers are only as good as the elasticity numbers behind them.

In particular, that .838 is sensitive to assumptions about how savings responds to the rate of return to savings.