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Group Welfare and the Formation of a Common Labor Market: Some Global Results

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Abstract

Consider a collection of isolated or autarkic regions. The original residents or natives of each region are by assumption a group with a welfare function defined over group members' consumption. Now suppose the regions form a common labor market and a federal government, and one type from each group can freely migrate to other regions. Under what circumstances is this change even potentially beneficial to all groups? We derive a necessary and sufficient condition that depends only on the exogenous parameters of our model. Earlier treatments of these issues focus on relationships among endogenous variables. Our condition underlies those relationships. We also show that there is nothing pathological about the conditions under which federalism must make some or all groups worse off. When it is possible to make all groups better off, we show that the change can be supported by Wildasin's (1991) corrected Nash equilibrium in redistributive transfers.

Keywords: federalism, factor mobility, redistribution, globalization, welfare economics

JEL Code: H10; H23; D60

1. Introduction

Consider a collection of isolated or autarkic regions. Suppose the original residents or natives of each region are a group with a welfare function defined over group members' consumption. Consumption is allocated to maximize group social welfare.

Now suppose the regions form a common labor market with a benevolent federal government. The mobile members of each group can work in any region, but they remain members of their group regardless of where they work. The federal government can move resources across the regions, and it can discriminate between the mobile and immobile agents within a region, but it cannot discriminate among workers on the basis of group identity. That is to say, migrants cannot be denied access to the local or federal transfers that are available to the native workers in a region. Our central question is, under what circumstances is this change beneficial to all groups?

To answer this question, we begin with the general normative issue it raises: under what circumstances is the creation of a common labor market even *potentially* beneficial to all groups? This is a question about the welfare possibilities under each regime. Under autarky, production in each region is determined by an initial allocation of inputs, and production and consumption must balance within each region. We use these conditions to define the set of feasible allocations for autarky and a welfare possibilities frontier. Under federalism, only aggregate production and consumption must balance, but given costless migration and nondiscrimination, all mobile agents in all regions must have the same level of consumption regardless of the group to which they belong. These conditions define the set of feasible allocations for federalism and a different welfare possibilities frontier.

Our main result characterizes, in terms of the exogenous parameters of the model, when it is possible for the shift to federalism to increase the welfare of all groups. Earlier treatments of these issues focus on relationships among endogenous variables. Our condition underlies those relationships. We also show that there is nothing pathological about the conditions under which federalism must make some or all groups worse off.

At its core, the analysis addresses a tradeoff that appears in many discussions of globalization. On the one hand, the common labor market ensures production efficiency. Aggregate output generally increases. On the other hand, each group loses most of its ability to influence the distribution of consumption within the group. All mobile agents must have the same level of consumption in all regions, which is a direct result of costless migration and nondiscrimination. Whether or not this change could benefit all groups depends fundamentally on the relative power of these two effects.

With the normative results established, we consider the question of when a shift to federalism will increase the welfare of all groups in equilibrium. Obviously, it would be futile to search for institutions that could achieve this result if the required allocations simply do not exist. The normative analysis provides a useful impossibility result for the positive analysis in this case. When such allocations do exist, we show that they can be supported by Wildasin's (1991) corrected Nash equilibrium in redistributive transfers.

The creation of common factor markets and the effect on local control are part of the large and growing literature on globalization.[1] Bolton and Roland (1997), Wildasin (1997), and Garrett (1998) discuss the general tradeoff between efficiency and local control, although they focus on more positive questions. Given our focus on local redistribution and group welfare, our work is most closely related to that of Wildasin (1991, 1994). Our approach and the general lessons are somewhat different, however. Wildasin (1994) shows that free migration cannot make all agents better off if, under autarky, the owners of immobile factors are taxed to provide transfer payments to mobile workers. Our focus is on explaining how certain exogenous parameters—the relative number of mobile agents, their importance in group welfare, and their productivity—simultaneously determine who is taxed under autarky and whether a shift to federalism makes all agents better off. While we use functional forms, we permit any finite number of countries with perhaps different amounts of fixed factors, and we completely characterize the relationship between autarky and federalism in this framework.

Section 2 presents the model and preliminary results. Section 3 states and explains our primary normative result, and Section 4 discusses equilibrium under federalism. We give brief conclusions in Section 5. The proofs of the key theorems are in the appendix.

2. Model and Preliminary Results

2.1. Feasible and Efficient Allocations

We assume production, preferences, and resource constraints are simple and of the form typically used to study common labor markets (Wildasin 1991). There are $n \geq 2$ regions, indexed by $i = 1, \ldots, n$. There are two types of agents, immobile (owners) and mobile (workers). We assume each region has the same number of immobile agents; in fact, we assume there is just one in each region, but the latter is for ease of notation only (see Section 2.4). Mobile agents provide a single unit of labor wherever they are allocated. The quantity of labor (or number of mobile agents) allocated to region i is denoted $l_i \geq 0$, and an allocation of labor is any vector $\mathbf{l} = (l_1, \ldots, l_n) \in \Re_+$.

The model we are developing derives from a literature that focuses on positive or equilibrium questions. In this literature, the mobile agents are "mobile" in the conventional sense (at some point they choose a region) and they are also "workers" (they sell labor to obtain income). The immobile agents are also "owners" (they rent assets to obtain income). It is essential to note that in the normative model we develop here, none of this holds. Individual endowments, incomes, and choices play no role in the analysis of feasibility, efficiency, and welfare possibilities. We use the conventional terms because they are convenient and because they are directly relevant in Section 4, when we turn to positive issues.

Region i is characterized by a production function, $f_i : \Re^1_+ \to \Re^1_+$, where $f_i(l_i)$ is the amount of consumption good produced by l_i units of labor. We specify this further in section 2.4. For the moment we assume that f_i is continuous on $[0, \infty)$, $f_i' > 0$ and $f_i'' < 0$ for all $l_i > 0$, and $\lim_{l_i \to 0} f_i'(l_i) = +\infty$.

The preferences of all agents depend only on the amount of consumption good they receive. The immobile agent in region i is allocated $y_i \geq 0$ units, so an allocation of consumption good to owners is $\mathbf{y} = (y_1, \ldots, y_n) \in \Re_+$. We assume that all workers allocated to the same region receive the same amount of the consumption good, $c_i \geq 0$. This is standard and allows us to denote an allocation of consumption good to workers by $\mathbf{c} = (c_1, \ldots, c_n) \in \Re_+$.

An *allocation* is any $(\mathbf{y}, \mathbf{c}, \mathbf{l}) \in \Re^{3n}_+$. The set of *feasible allocations* is:

$$X \equiv \left\{ (\mathbf{y}, \mathbf{c}, \mathbf{l}) \in \Re^{3n}_+ \,\middle|\, \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} l_i c_i \leq \sum_{i=1}^{n} f_i(l_i); \right.$$
$$\left. \mathbf{y} \gg 0, \mathbf{c} \gg 0, \mathbf{l} \gg 0; \sum_{i=1}^{n} l_i = l \right\}$$

We assume as part of the definition of feasibility that every region is assigned some workers and all consumers receive some consumption good. This is standard and simplifies the formal analysis.[2]

An allocation is efficient if it is feasible and there is no other feasible allocation that provides all agents with at least as much consumption good and some agents with strictly more. If $(\mathbf{y}, \mathbf{c}, \mathbf{l})$ is efficient then $\mathbf{l}$ must maximize $\sum_{i=1}^{n} f_i(l_i)$ subject to $\sum_{i=1}^{n} l_i = l$ and $l_i > 0$, all i. If not, there would be a feasible reallocation of labor that provided strictly more output, and all agents could be given more consumption good. Our assumptions on technology guarantee that the maximum of total output subject to $\sum_{i=1}^{n} l_i = l$ exists and occurs at a unique and interior allocation of labor. Denote this allocation $\mathbf{l}^* = (l_1^*, \ldots, l_n^*)$. Then the set of efficient allocations may be written:

$$\mathcal{E} \equiv \left\{ (\mathbf{y}, \mathbf{c}, \mathbf{l}^*) \in \Re_+^{3n} \;\middle|\; \sum_{i=1}^{n} y_i + \sum_{i=1}^{n} l_i^* c_i = \sum_{i=1}^{n} f_i(l_i^*); \quad \mathbf{y} \gg 0, \mathbf{c} \gg 0 \right\}$$

Thus, an allocation is efficient as long as output is maximized and all output is consumed.

2.2. Allocations Under Autarky and Federalism

Autarky is a condition of self-sufficiency. This requires consumption within each region to be less than or equal to production in that region. To assess welfare possibilities under autarky, we want to restrict attention to feasible allocations with this property.

We further assume that all regions have the same number of workers under autarky. This assumption is not quite as strong as it appears, since we do not assume that the regions have identical technologies. Its role in the analysis becomes clear below, when we define group welfare. We impose it now so we can incorporate all of the exogenous features of autarky into our definition of feasibility under autarky. The set of *feasible allocations under autarky*, denoted X^A, is:

$$X^A \equiv \{(\mathbf{y}, \mathbf{c}, \mathbf{l}) \in X \mid y_i + l_i c_i \leq f_i(l_i),\ l_i = l/n,\ \text{all } i\}$$

Given the discussion of efficiency above, we know that no element of X^A is efficient unless a uniform allocation of labor maximizes total output. This is unlikely to occur. Below we impose an assumption that guarantees that it never occurs, and so all feasible allocations under autarky will be inefficient.

To motivate our definition of the feasible allocations under federalism, we briefly digress to a positive model. Individuals in federal systems are citizens of both a region and the nation. We assume that workers have the right to work anywhere in the nation, and migrants cannot be distinguished from native workers by either the private sector or the regional or national public sectors. Thus, there is no discrimination by any sector on the basis of group identity. This implies that the net wage of workers in any region, which is their income after all taxes and transfers from all levels of government (and which also equals their consumption), is the same in all regions. We say that a group of regions form a *common labor market* if there is costless migration coupled with nondiscrimination. Note that it is still possible for the public sectors to discriminate between workers and owners within a region and for fiscal policies to differ across regions.

With the previous discussion as motivation, we return to our normative model and define the set of *feasible allocations under federalism*, denoted X^F, as the feasible allocations in which the consumption of all workers in all regions is the same:

$$X^F \equiv \{(\mathbf{y}, \mathbf{c}, \mathbf{l}) \in X \mid c_i = c_{i'},\ \text{all } i, i'\}$$

Two results are immediate from the previous definitions. First, X^F contains efficient allocations. Specifically, $X^F \cap \mathcal{E}$ consists of all feasible allocations in which output is maximized, all output is consumed, and the consumption of all workers in all regions is the same. Second, any equilibrium with a common labor market (and with all regions occupied by workers) leads to an alloc1tion in X^F.

2.3. Group Welfare

We suppose that the original residents or natives of each region are a group whose social welfare depends on the consumption of its members. Given our previous assumptions, all groups have $1+(l/n)$ members.[3] We refer to region i as group i's "homeland." Group membership is fixed, so group welfare depends on the consumption of the mobile agents regardless of where they are allocated.[4]

While it is essential to distinguish *regional* production from *group* welfare, we do not need to keep track of both region and group in the notation for consumption. Under autarky, all members of all groups consume in their respective homelands. So, only one subscript on consumption is needed. Under federalism, the immobile agents continue to consume in their homelands. The mobile agents may now consume outside their homelands, but they consume the same amount regardless of their group or their location, so no subscript is needed at all. Thus, the welfare of group i in either case may be unambiguously denoted by a function $v_i : \Re^2_+ \rightarrow \Re^1_+$, written $v_i(y_i, c_i)$. The number of individuals in the group is suppressed since it is the same for all groups.

2.4. Functional Forms

We assume that group welfare has a generalized Bernoulli-Nash functional form:[5]

$$v_i(y_i, c_i) = a_i y_i^{\alpha} c_i^{1-\alpha}, \quad a_i > 0,\ 0 < \alpha < 1, \ \text{ all } i$$

For example, suppose group welfare equals the product of individual incomes, so $v_i' = y_i c_i^{l/n}$. If $\alpha = 1/[1 + (l/n)]$ then v_i is just a monotone transformation of v_i'. Suppose group welfare equals the average of log income, so $v_i' = [\ln y_i + (l/n)\ln c_i]/[1 + (l/n)]$. Again, v_i is a monotone transformation of v_i'. We do not place these restrictions on α, but they suggest that it should be thought of as relatively small.[6]

We assume that production takes the form:

$$f_i(l_i) = t_i l_i^{\beta}, \quad t_i > 0, \ \ 0 < \beta < 1, \ \text{ all } i$$

Earlier we observed that we should expect production to be inefficient under autarky since all regions have the same number of workers. Given this functional form, production is efficient under autarky if and only if $t_i = t_{i'}$ for all i, i'. For simplicity, we assume that this does *not* hold. Formally, let $\mathbf{I}$ denote the set of vectors in $\Re^n_{++}$ in which all terms are the same. Define the vector $\mathbf{t} = (t_1, \ldots, t_n)$. We assume that $\mathbf{t} \in \Re^n_{++} \setminus \mathbf{I}$.

2.5. Group Welfare Possibilities Under Autarky and Federalism

It is convenient to refer to autarky and federalism as *regimes*. For regimes $k = A, F$, the *regime welfare possibilities set* is:[7]

$$V^k \equiv \{\mathbf{v} \in \Re^n \mid \mathbf{v} \gg 0 \text{ and there exists } (\mathbf{y}, \mathbf{c}, \mathbf{l}) \in X^k \text{ such that } v_i \le v_i(y_i, c_i) \text{ for all } i\}$$

The *regime frontier* is:

$$\mathcal{V}^k \equiv \{\mathbf{v} \in \Re^n \mid \mathbf{v} \in V^k \text{ and there is no } \tilde{\mathbf{v}} \in V^k \text{ such that } \tilde{\mathbf{v}} > \mathbf{v}\}$$

Theorems 1 and 2 characterize the autarky and federalism welfare possibilities sets and frontiers. Note that the functional forms for f_i and v_i require $2n + 2$ parameters: a value of a_i and t_i for each region plus α and β. Define the constant:

$$\gamma \equiv \alpha + \beta - 1$$

The theorems will always assume that the parameters belong to the set:

$$P \equiv \{(\mathbf{a}, \mathbf{t}, \alpha, \beta) \mid \mathbf{a} \in \Re^n_{++},\ \mathbf{t} \in \Re^n_{++} \setminus \mathbf{I},\ 0 < \alpha < 1,\ 0 < \beta < 1;\ \gamma \neq 0\}$$

Notice that we do not allow all of the regions to be the same size and we require $\gamma \neq 0$. These are minor technical assumptions that simplify the analysis.

Theorem 1 (Autarky)**.** *Fix* $\mathbf{p} \in P$. *Define*:

$$v_i^A \equiv \alpha^\alpha (1-\alpha)^{1-\alpha} a_i t_i (l/n)^\gamma$$

Then

(a) $V^A = \{\mathbf{v} \in \Re^n \mid \mathbf{v} \gg 0,\ v_i \le v_i^A,\ \ \textit{all } i\}$
(b) $\mathcal{V}^A = \{(v_1^A, \ldots, v_n^A)\}$

Theorem 2 (Federalism)**.** *Fix* $\mathbf{p} \in P$. *Define*:

$$Y \equiv \left\{\mathbf{y} \in \Re^n \mid \mathbf{y} \gg 0,\ \sum_{i=1}^n y_i = \alpha \sum_{i=1}^n f_i(l_i^*)\right\}$$

where

$$l_i^* = \left(\frac{l}{\tau}\right) t_i^{\frac{1}{1-\beta}},\quad \tau \equiv \sum_{i=1}^n t_i^{\frac{1}{1-\beta}},\quad v_i^F(y_i) \equiv a_i y_i^\alpha (c^F)^{1-\alpha},\quad c^F \equiv (1-\alpha)\left(\frac{\tau}{l}\right)^{1-\beta}$$

Then

(a) $V^F = \left\{\mathbf{v} \in \Re^n \mid \mathbf{v} \gg 0 \textit{ and there exists } \mathbf{y} \in Y \textit{ such that } v_i \le v_i^F(y_i) \textit{ for all } i\right\}$
(b) $\mathcal{V}^F = \left\{\mathbf{v} \in \Re^n \mid \textit{there exists } \mathbf{y} \in Y \textit{ such that } v_i = v_i^F(y_i) \textit{ for all } i\right\}$

The proofs of these theorems follow readily from standard results in concave programming (Takayama, 1974) and the definitions of the various sets.[8]

Theorem 2 shows that there is a simple way to generate the federalism frontier. The allocation of labor at all points on the frontier is $(l_1^*, \ldots, l_n^*)$. This maximizes total output (and equalizes the marginal product of labor) across regions.[9] In addition, at all points on the frontier, a fixed α share of total output is allocated to owners and the remainder to workers. Given this, the frontier is determined by varying the distribution of resources to owners within the set Y.[10]

3. Comparing Welfare Possibilities

3.1. Main Results

Intuitively, a shift from autarky to federalism creates an important tradeoff. On the one hand, labor is no longer allocated inefficiently. On the other hand, each group loses control over the distribution of income between its immobile and mobile members. Our central question is, under what circumstances is this change even potentially beneficial to all groups? Recall that all of the allocations on the federalism frontier are efficient while none of the allocations on the autarky frontier are efficient. Thus, we are especially interested in situations in which a move from the autarky to the federalism frontier—an efficiency enhancing change—must make one or all groups worse off.

In order to state our main result, we need one more definition. Given a particular parameter vector $\mathbf{p} \in P$ and two regimes j and k, we say that *regime j dominates regime k at* $\mathbf{p}$ if (a) for all $\mathbf{v}^j \in \mathcal{V}^j$ and $\mathbf{v}^k \in \mathcal{V}^k$ there is some group i such that $v_i^j > v_i^k$, and (b) there exists $\mathbf{v}^j \in \mathcal{V}^j$ and $\mathbf{v}^k \in \mathcal{V}^k$ such that $v_i^j \geq v_i^k$ for all groups i. Part (a) ensures that no point on the k frontier is to the northeast of any point on the j frontier, and part (b) ensures that some point in the j frontier is to the northeast of some point on the k frontier.[11]

Our main result is:

Theorem 3. *Given any* $\mathbf{p} \in P$:

(a) *If* $\gamma < 0$ *then autarky dominates federalism.*
(b) *If* $\gamma > 0$ *then federalism dominates autarky.*

This is a "global" result in at least three senses. First, it holds for all of P and not just for specific numerical values of the parameters. Second, the dominance relation is complete: it holds one way or the other at each $\mathbf{p} \in P$. Finally, the only parameters in $\mathbf{p}$ that are relevant are those determining γ (namely α and β). Thus, dominance is not sensitive to the relative sizes of the jurisdictions. Whichever regime is dominant remains so for every $\mathbf{t} \in \Re^n_{++} \setminus \mathbf{I}$.

We use the following elementary result in our discussion below:

Theorem 4. *Let c_i^A denote worker consumption in region i under autarky. Given any* $\mathbf{p} \in P$, $c_i^A < c^F < c_{i'}^A$ *if and only if* $t_i^{1/(1-\beta)} < \tau/n < t_{i'}^{1/(1-\beta)}$.

3.2. Discussion

The parameter γ will be negative if the number of mobile agents is large relative to the number of immobile agents (so α is near zero) and total output is mostly independent of how labor is allocated (so β is near zero). When these conditions hold, each group's welfare depends mostly on the welfare of its mobile agents and total output is near its maximum regardless of how labor is allocated. A shift to federalism can produce only a modest increase in total output, while mobile agents from "large" homelands must see their consumption fall (Theorem 4). To preserve the welfare of a group from a large homeland, either the common level of worker consumption (c^F) must be similar to the original consumption level for these workers, or the owner's consumption must increase enough to offset its small weight in group welfare. Given the small quantity of resources that are newly available, it is not possible to do either without lowering the welfare of some other group.

While there are not enough extra resources to make all groups better off and give all workers the same level of consumption, extra resources are available. It would therefore be possible to make all groups better off if not for the restriction that all workers consume the same amount. This restriction, which is motivated by costless migration and nondiscrimination, amounts to a limitation on the ability of the group to redistribute its resources. Overall, then, it is the loss of group control, and not the small increase in aggregate resources, that is the fundamental source of the welfare loss under federalism. To see the issue most clearly, we illustrate the case in which all groups are actually allocated more resources under federalism yet all groups are worse off. The welfare of all groups could increase if they could freely reallocate the resources of their members. The requirement that all workers have the same level of consumption rules this out.

Figure 1 presents the welfare possibilities frontiers under autarky and federalism with two groups (and regions). It also specifies a particular distribution of welfare on the federalism frontier.[12]

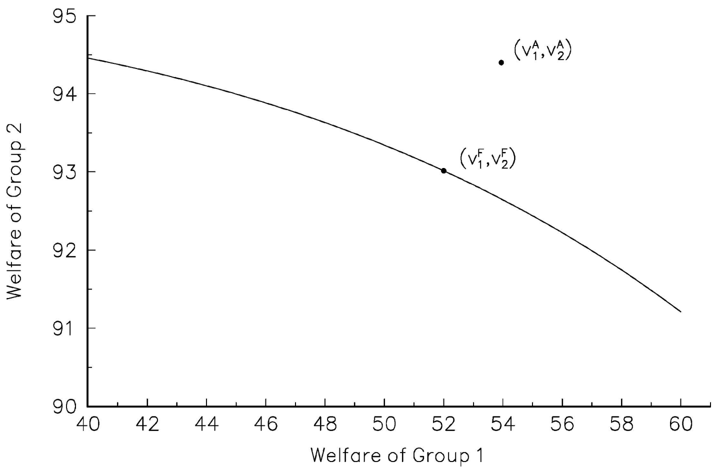

Figure 1. Welfare possibilities frontiers ($\gamma < 0, t_2 > t_1$).

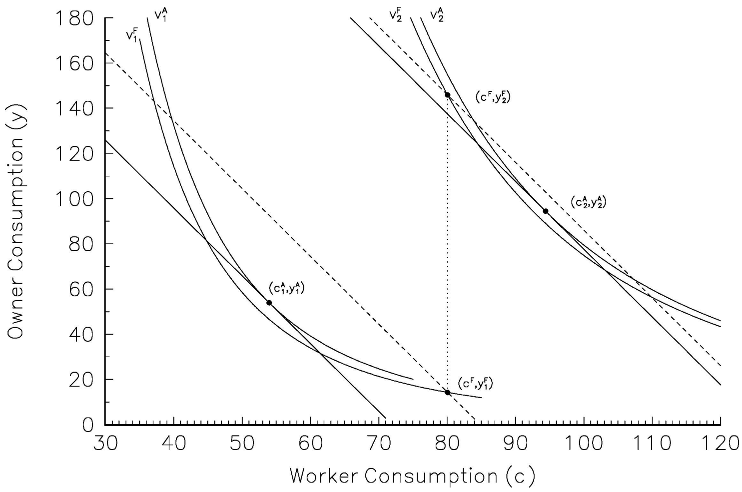

Figure 2. Consumption allocations ($\gamma < 0, t_2 > t_1$).

Figure 2 takes these four levels of group welfare and presents the distributions of income and the indifference curves associated with them. Under autarky, for each group, income is allocated to maximize its welfare subject to the production possibilities of its homeland, taking the allocation of labor as given. Those production possibilities (which are also consumption possibilities) are indicated by the solid lines.[13]

Under federalism, incomes are allocated within one group to maximize its welfare subject to aggregate production possibilities, a specified level of welfare for the other group, and the requirement that all mobile agents receive the same income. In Figure 2, the dashed lines indicate the distributions of income that each group could achieve if it could freely redistribute the total resources it receives at the optimum. Note that they are both further to the right than the corresponding solid lines.[14]

Figure 2 verifies the explanation for Theorem 3 offered above. Under federalism, mobile agents from the small homeland obtain higher income while mobile agents from the large homeland obtain less. Income is also shifted away from the immobile agent in the small homeland.

In this example, both groups have greater total income available to them under federalism than under autarky. The fact that all mobile agents must receive the same income is a binding constraint, in the sense that both groups could achieve higher welfare if they could freely reallocate their resources.

More generally, this result illustrates an equity/efficiency tradeoff. Given any $\mathbf{p} \in P$, the allocations giving the point on the autarky frontier are not Pareto optimal. Those giving the points on the federalism frontier are. In this sense a move to federalism is always efficiency enhancing. In the case $\gamma < 0$, however, consumption by certain individuals falls and the welfare of one or more groups falls. The gains in output from production efficiency need not trump the negative effects of costless migration and nondiscrimination on the distribution of consumption and group welfare.

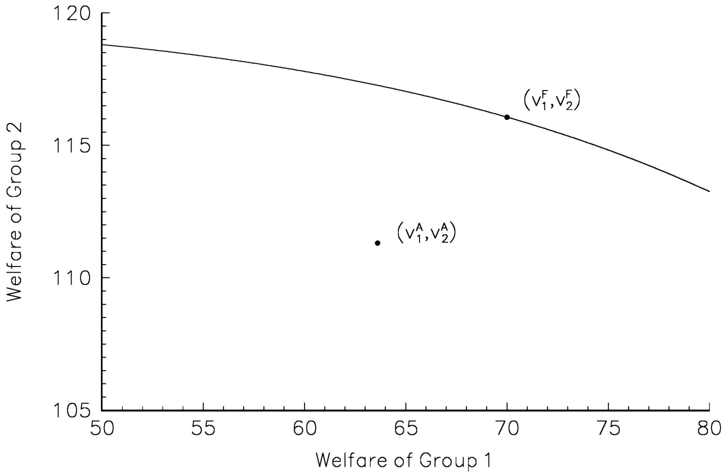

Figure 3. Welfare possibilities frontiers ($\gamma > 0, t_2 > t_1$).

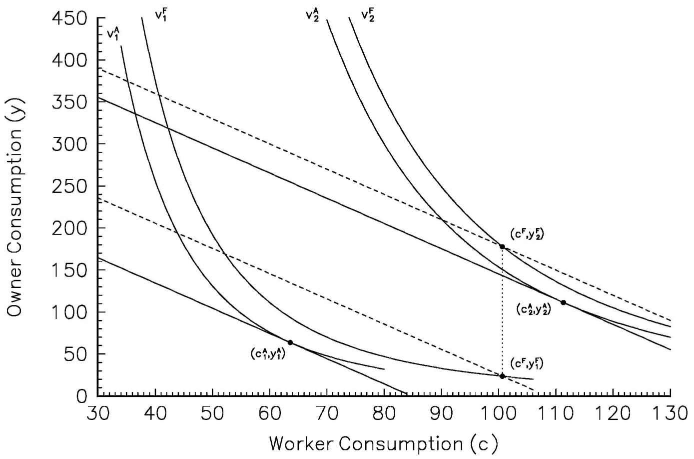

Figure 4. Consumption allocations ($\gamma > 0, t_2 > t_1$).

Figures 3 and 4 are analogous.[15] Notice that, as in Figure 2, the dashed lines in Figure 4 are to the right of the corresponding solid lines.[16] In this case, however, federalism makes both groups better off.

4. Equilibrium on the Frontiers

4.1. Transfers Under Autarky and Regime Dominance

Under autarky, each worker is paid his marginal product and a per-worker intra-regional transfer z_i. Whether the transfer is positive or negative is addressed below. Net income of workers in region i is then:

$$c_i = f_i'(l_i) + z_i \tag{1}$$

Owners receive all of the income from fixed factors, pay workers their marginal product, and fund the transfer. Net income of owners in region i is:

$$y_i = f_i(l_i) - l_i f_i'(l_i) - z_i l_i \tag{2}$$

In both cases, $l_i = l/n$.

Substituting (1) and (2) into the group welfare function and maximizing with respect to the transfer, we obtain $z_i = -\gamma t_i (n/l)^{1-\beta}$. If we do this for each group, the distribution of group welfare is the point on the autarky frontier.

More interesting, however, is the relationship between the direction of the transfer and regime dominance. If $\gamma < 0$, then workers are subsidized, owners are taxed, and federalism cannot make all groups better off. This is consistent with the main finding in Wildasin (1994).[17] The correlation between the direction of transfers and regime dominance is both intuitive and empirically interesting. It is not a complete explanation of either, however. We expect, for example, that the productivity gains from federalism should play a role in determining regime dominance. As our results make clear, this is a critical factor in determining both the direction of transfers under autarky and whether a shift to federalism makes all agents better off.

4.2. Equilibrium on the Federalism Frontier

Our assumptions about production, preferences, labor supply and group welfare are special cases of those in Wildasin (1991). Actually, Wildasin initially assumes that the owner in each region is altruistic in the sense that her preferences depend on worker income in addition to her own. Wildasin notes (p. 766), however, that all of his results go through if the owner derives utility from just her own income but policy in region i is chosen to maximize a group welfare function defined over y_i and c. We can therefore make use of his equilibrium analysis, and our welfare analysis has implications for his results.[18]

In the Wildasin model, the central government levies lump sum taxes T_i on the owner in each region and subsidizes intra-regional transfers between workers and owners at rate s_i. Equation (1) is unchanged, but equation (2) becomes:

$$y_i = f_i(l_i) - l_i f_i'(l_i) - (1 - s_i) z_i l_i - T_i \tag{3}$$

The central government must also balance its budget:

$$\sum_{i=1}^{n} (s_i z_i l_i - T_i) = 0 \tag{4}$$

The central and regional governments make their choices simultaneously, after which workers move. Worker migration creates a common level of worker consumption, c, that is contingent on s_i, z_i and T_i, $i = 1, \ldots, n$. Each regional government chooses z_i to maximize the welfare of the group from that region. It takes all of the other transfer variables as given, but it recognizes the impact that its choice of z_i has on the number of workers in region i and on the (common) level of worker consumption. It takes no account of the fact that changes in c affect the welfare of other groups. Wildasin establishes that this creates an externality. By assumption, the goal of the central government is to internalize this externality. A "corrected Nash equilibrium in redistributive transfers" is then a vector of subsidies s_i and transfers z_i, $i = 1, \ldots, n$ such that the transfers form a Nash equilibrium and the marginal external effect of the transfers is zero for each region.[19]

We now have the following:

Theorem 5. *A corrected Nash equilibrium in redistributive transfers achieves a distribution of group welfare on the federalism frontier.*

The proof uses Propositions 2 and 3 in Wildasin (1991) to show that the equilibrium satisfies all of the conditions in Theorem 2 above.

4.3. Achieving the Benefits of Federalism When $\gamma > 0$

When federalism dominates autarky, it is possible to make all groups better off by changing regimes. The previous analysis shows, however, that a federal government must undertake two distinct tasks in order to actually improve the welfare of all groups. First, it must choose taxes and transfers in order to internalize the fiscal externalities created by decentralized intraregional transfers. If not, the equilibrium distribution of welfare will not be on the federalism frontier. Second, it must use the right combination of lump-sum taxes to achieve the right point on the federalism frontier. If not, the equilibrium distribution of welfare need not make all groups better off. It is fair to say that the common factor market must be carefully governed by the higher tier government for the benefits of federalism to be realized for all groups.[20]

4.4. The Irrelevance of Fiscal Institutions When $\gamma < 0$

When autarky dominates federalism, our previous analysis shows that internalizing fiscal externalities is somewhat beside the point. Creating the common factor market must make some or all groups worse off whether the fiscal externality is internalized or not.

More generally, Theorem 3 answers the question of whether there exist *any* fiscal institutions that could increase welfare for all groups in equilibrium. Given a common factor market, so all workers obtain the same net wage, there are no such institutions. The allocations that could achieve this goal simply do not exist.

Of course, it is not always possible for regions to maintain control over their borders. Wildasin's result is always relevant if, in his words, "direct control over the level of migration or over the access of migrants to the benefits of redistributive policies is infeasible."[21] When autarky dominates federalism, however, group welfare provides a rationale for some or all regions to try to maintain control over their borders.

4.5. The Basis for Redistribution

Finally, it is clear that the assumption that regions redistribute on the basis of residence and not group membership is critical. Consider the opposite extreme, in which regional governments have no power to tax or subsidize residents per se. Instead, regional governments redistribute income among members of the group regardless of where they reside. Production is efficient, since location has no effect on net wages except through gross wages. Workers from all groups therefore seek the highest gross wage when they choose locations, and this equalizes the marginal product of labor. Costless migration does not create a common level of worker consumption. The groups are strategically independent, since redistribution by each group has no direct or indirect effects on the incomes of workers or owners in any other group. There are no fiscal externalities. Lump-sum transfers across groups may still be needed to make sure that the welfare of all groups increases after migration, but that would be the only role for a federal government.[22]

5. Conclusion

We find that autarky dominates federalism if the number of mobile agents is large relative to the number of immobile agents and total output is mostly independent of how labor is allocated. In this case, a shift to federalism produces only a modest increase in total output. Mobile agents from large homelands must see their consumption fall as a common level of consumption is established. To preserve the welfare of a group from a large homeland, either the common level of worker consumption must be similar to the original consumption level for these workers, or the owner's consumption must increase enough to offset its small weight in group welfare. Given the small quantity of resources that are newly available, it is not possible to do either without lowering the welfare of some other group.

Since extra resources are available, it would be possible to make all groups better off if not for the restriction that all workers consume the same amount. The common level of consumption amounts to a constraint on the ability of the group to redistribute the resources of its members. When autarky dominates federalism, the tradeoff between more output and less local control over distribution works against one or more groups. Earlier analyses of this tradeoff focus on relationships among endogenous variables, like the direction of transfers and the effects of migration on incomes. Our condition, which is defined on the parameters of the model, underlies those relationships.

The normative analysis that we start with provides a useful impossibility result for the positive analysis that follows. It would be futile to search for institutions that could make all groups better off if the required allocations simply do not exist. When such allocations do exist, we show that they can be supported by Wildasin's (1991) corrected Nash equilibrium in redistributive transfers. A federal government must undertake two distinct tasks in order to actually improve the welfare of all groups. First, it must choose taxes and transfers in order to internalize the fiscal externalities created by decentralized intraregional transfers. Second, it must use the right combination of lump-sum taxes to achieve the right point on the federalism frontier. It is fair to say that the common factor market must be carefully governed by the higher tier government for the benefits of federalism to be realized for all groups.

Finally, we derive strong conclusions within a specific and tractable framework. It would be useful to know whether our basic qualitative results hold up in a more general setting. It is likely that our dominance relation is not complete in a more general model, so one could construct examples in which neither regime dominates the other. We suspect, however, that our qualitative results are general because our characterizations of federalism and autarky are general and so is our basis for comparing them. In any framework, the move from autarky to federalism creates a tradeoff between production efficiency and local control. We therefore expect quite generally that federalism must make some groups worse off when it offers few benefits in terms of extra output and high costs in terms of the loss of local control, however these are formalized.

Appendix

The proofs of Theorems 1–2 and of the Lemma below are contained in the supplemental appendix (the proof of the Lemma uses Hölder's inequality).

Lemma. *Suppose* $\mathbf{p} \in P$.

$$\gamma < 0 \Rightarrow \sum_{i=1}^{n} t_i^{\frac{1}{\alpha}} > n^{\frac{\gamma}{\alpha}} \tau^{\frac{1-\beta}{\alpha}} \tag{A1}$$

$$\gamma > 0 \Rightarrow \sum_{i=1}^{n} t_i^{\frac{1}{\alpha}} < n^{\frac{\gamma}{\alpha}} \tau^{\frac{1-\beta}{\alpha}} \tag{A2}$$

Proof of Theorem 3: (a). Suppose the first condition in the definition of dominance fails. Then there exists $\mathbf{v}^F \in \mathcal{V}^F$ such that for all i, $v_i^A \leq v_i^F$ (recall there is only one vector $\mathbf{v}^A$ in $\mathcal{V}^A$). It follows from Theorems 1 and 2 that there exists $\mathbf{y} \in Y$ such that for all i:

$$v_i^A = \alpha^\alpha (1-\alpha)^{1-\alpha} a_i t_i (l/n)^\gamma \leq a_i y_i^\alpha \left[(1-\alpha) \left(\frac{\tau}{l} \right)^{1-\beta} \right]^{1-\alpha} = v_i^F, \quad i = 1, \ldots, n$$

If we eliminate the exponent on y_i, sum over all i and use the fact $\sum_{i=1}^n y_i = \alpha \sum_{i=1}^n f_i(l_i^*) = \alpha l^\beta \tau^{1-\beta}$, we obtain $\sum_{i=1}^n t_i^{\frac{1}{\alpha}} \leq n^{\frac{\gamma}{\alpha}} \tau^{\frac{1-\beta}{\alpha}}$. This contradicts (A1).

To establish the second condition in the definition of dominance, we construct $\mathbf{v}^F \in \mathcal{V}^F$ such that for all i, $v_i^A > v_i^F$. Define $\tilde{\mathbf{y}}_i$ to solve:

$$v_i^A = \alpha^\alpha (1-\alpha)^{1-\alpha} a_i t_i (l/n)^\gamma = a_i \tilde{y}_i^\alpha (c^F)^{1-\alpha}, \quad i = 1, \ldots, n \tag{A3}$$

where c^F is defined in Theorem 2. Explicitly:

$$\tilde{y}_i = \alpha \tau^{\frac{(1-\beta)(\alpha-1)}{\alpha}} l^\beta n^{-\frac{\gamma}{\alpha}} t_i^{\frac{1}{\alpha}}, \quad i = 1, \ldots, n$$

We next define $\hat{\mathbf{y}}_i$ by rescaling each $\tilde{y}_i$:

$$\hat{y}_i = \tilde{y}_i \left(\frac{\alpha l^\beta \tau^{1-\beta}}{\sum_{i=1}^n \tilde{y}_i} \right), \quad i = 1, \ldots, n \tag{A4}$$

Clearly $\hat{\mathbf{y}} \in Y$, since $\hat{y}_i > 0$ for all i and $\sum_{i=1}^n \hat{y}_i = \alpha l^\beta \tau^{1-\beta} = \alpha \sum_{i=1}^n f_i(l_i^*)$. Finally, define:

$$v_i^F \equiv a_i \hat{y}_i^\alpha (c^F)^{1-\alpha}, \quad i = 1, \ldots, n$$

We have $\mathbf{v}^F \in \mathcal{V}^F$ by Theorem 2.

We now show that the scaling factor in (A4) is strictly less than 1. We have:

$$\begin{aligned}\sum_{i=1}^n \tilde{y}_i &= \alpha \tau^{\frac{(1-\beta)(\alpha-1)}{\alpha}} l^\beta n^{-\frac{\gamma}{\alpha}} \sum_{i=1}^n t_i^{\frac{1}{\alpha}} \\ &> \alpha \tau^{\frac{(1-\beta)(\alpha-1)}{\alpha}} l^\beta \tau^{\frac{1-\beta}{\alpha}} \\ &= \alpha l^\beta \tau^{1-\beta}\end{aligned}$$

where the strict inequality comes from (A1). It follows that $\hat{y}_i < \tilde{y}_i$ for all i. This with (A3) completes the proof:

$$v_i^A = \alpha^\alpha (1-\alpha)^{1-\alpha} a_i t_i (l/n)^\gamma = a_i \tilde{y}_i^\alpha (c^F)^{1-\alpha} > a_i \hat{y}_i^\alpha (c^F)^{1-\alpha} = v_i^F, \quad i = 1, \ldots, n$$

(b) The proof is the same as the proof of (a), except we use (A2) and reverse the inequalities. □

Proof of Theorem 4: Maximizing group welfare in region i under autarky gives $c_i^A = (1-\alpha) t_i (l/n)^{\beta-1}$. Using the formula for c^F in Theorem 2 and rearranging gives the result. □

Proof of Theorem 5: We are given a corrected Nash equilibrium in redistributive transfers. This determines values for the transfer instruments, an equilibrium allocation $(\mathbf{y}, \mathbf{c}, \mathbf{l})$, and a distribution of group welfare. Wildasin establishes that the intra-regional transfer z_i is the same in each region (Proposition 2) and the following "Samuelson" condition holds at the allocation (Proposition 3):

$$\sum_{i=1}^n \frac{\partial v_i / \partial c}{\partial v_i / \partial y_i} = l$$

We use these results as follows. Migration implies that worker net income, $f_i'(l_i) + z_i$, is the same everywhere. This with Proposition 2 implies that worker gross income (the marginal product of labor) is identical everywhere. Production is therefore efficient, so total output is $\sum_{i=1}^n f_i(l_i^*)$. This with (1),(3),(4) summed over all i and $c_i = c$ for all i give $\sum_{i=1}^n y_i + lc = \sum_{i=1}^n f_i(l_i^*)$. The Samuelson condition, given our functional forms, gives $[(1-\alpha)/\alpha] \sum_{i=1}^n y_i = lc$. These last two equations can be used to show that $\mathbf{y} \in Y$ and the common level of worker consumption is c^F, defined in Theorem 2. It follows that the distribution of group welfare is on the federalism frontier. □

Supplemental Appendix

Lemma 1. *Let $l_i \equiv l/n$. For all i, the unique pair $(y_i^A(l_i), c_i^A(l_i))$ for which $v_i(y_i, c_i)$ achieves a global maximum on $\Re_{++}^2$ subject to $f_i(l_i) \geq y_i + l_i c_i$ is:*

$$y_i^A(l_i) = \alpha t_i l_i^\beta, \quad c_i^A(l_i) = (1-\alpha) t_i l_i^{\beta-1}$$

As an immediate corollary, for all $(y_i, c_i) \in \Re^2_{++}$ *satisfying* $f_i(l_i) \geq y_i + l_i c_i$:

$$v_i^A(l_i) \equiv \alpha^\alpha (1-\alpha)^{1-\alpha} a_i t_i l_i^{\alpha+\beta-1} = v_i\left[y_i^A(l_i), c_i^A(l_i)\right] \geq v_i(y_i, c_i)$$

Proof of Lemma 1: Available on request. □

Proof of Theorem 1: Available on request. □

Lemma 2. *Fix any distribution of welfare* $\mathbf{v} \in V^F$. *Fix an arbitrary region; for notational convenience, region* n. *The unique vector* $(\mathbf{y}^F(\mathbf{v}), \mathbf{c}^F, \mathbf{l}^F,)$ *for which* $v_n(y_n, c_n)$ *achieves a global maximum subject to* $v_i(y_i, c_i) \geq v_i$, $i = 1, \ldots, n-1$ *and* $(\mathbf{y}, \mathbf{c}, \mathbf{l}) \in X^F$ *is*:

$$y_i^F(\mathbf{v}) = \left(\frac{v_i}{a_i}\right)^{\frac{1}{\alpha}} \left[(1-\alpha)\left(\frac{\tau}{l}\right)^{1-\beta}\right]^{-\frac{1-\alpha}{\alpha}}, \quad i = 1, \ldots, n-1$$

$$y_n^F(\mathbf{v}) = \alpha l^\beta \tau^{1-\beta} - \sum_{i=1}^{n-1} y_i^F(\mathbf{v})$$

$$c_i^F = (1-\alpha)\left(\frac{\tau}{l}\right)^{1-\beta}, \quad i = 1, \ldots, n$$

$$l_i^F = \left(\frac{l}{\tau}\right) t_i^{\frac{1}{1-\beta}}, \quad i = 1, \ldots, n$$

where τ is defined in the statement of Theorem 2. Furthermore, define:

$$v_i^F(y_i) \equiv a_i y_i^\alpha (c^F)^{1-\alpha}$$

An immediate corollary is $v_i^F[y_i^F(\mathbf{v})] = v_i[y_i^F(\mathbf{v}), c_i^F] = v_i$ for $i = 1, \ldots, n-1$ and $v_n^F[y_n^F(\mathbf{v})] = v_n[y_n^F(\mathbf{v}), c_n^F] \geq v_n$, where c^F is defined in the statement of Theorem 2.

Proof of Lemma 2: Available on request □

Proof of Theorem 2: (a) Define:

$$W^F \equiv \left\{\mathbf{v} \in \Re^n | \mathbf{v} \gg 0, (\exists \mathbf{y} \in Y)\left(v_i \leq v_i^F(y_i), \text{ all } i\right)\right\}$$

where $v_i^F(y_i)$ is given in (B5). The theorem claims that $V^F = W^F$.

Fix any vector $\hat{\mathbf{v}} \in V^F$. Using $\hat{\mathbf{v}}$ in the maximization problem in Lemma 2 gives $(\mathbf{y}^F(\hat{\mathbf{v}}), \mathbf{c}^F, \mathbf{l}^F)$. By construction (see the statement of the Lemma) we obviously have $\mathbf{y}^F(\hat{\mathbf{v}}) \in Y$. It is a conclusion of the Lemma that $\hat{v}_i \leq v_i^F[y_i^F(\hat{\mathbf{v}})]$ for all i. Therefore $\hat{\mathbf{v}}$ is in W^F.

Now fix any vector $\hat{\mathbf{v}} \in W^F$. By definition then there exists $\hat{\mathbf{y}}$ such that $\hat{\mathbf{y}} \gg 0$, $\sum_{i=1}^n \hat{y}_i = \alpha l^\beta \tau^{1-\beta}$ and $\hat{v}_i \leq v_i^F(\hat{y}_i)$ for all i. Define $\hat{c}_i \equiv c_i^F$ and $\hat{l}_i \equiv l_i^F$ for all i. This defines the vector $(\hat{\mathbf{y}}, \hat{\mathbf{c}}, \hat{\mathbf{l}})$.

By definition, the set V^F consists of all $\mathbf{v} \in \Re^n$ such that (i) $\mathbf{v} \gg 0$, and for which there exists $(\mathbf{y}, \mathbf{c}, \mathbf{l}) \in \Re^{3n}$ such that (ii) $(\mathbf{y}, \mathbf{c}, \mathbf{l}) \gg 0$, (iii) $\sum_{i=1}^n y_i + \sum_{i=1}^n l_i c_i \leq \sum_{i=1}^n f_i(l_i)$, (iv) $c_i = c_{i'}$ for all i, i', (v) $\sum_{i=1}^n l_i = l$, and (vi) $v_i(y_i, c_i) \geq v_i$ for all i.

Property (i) holds by definition of W^F and (ii) and (iv) hold by construction. For (iii), one can directly verify that $\sum_{i=1}^n f_i(\hat{l}_i) = l^\beta \tau^{1-\beta}$ and $\sum_{i=1}^n \hat{y}_i + l\hat{c} = \alpha l^\beta \tau^{1-\beta} + l(1-\alpha)(\frac{\tau}{l})^{1-\beta} = l^\beta \tau^{1-\beta}$. Similarly for (v). Finally, the definition of W^F gives $\hat{v}_i \leq v_i(\hat{y}_i, \hat{c}_i)$ for all i, which is (vi). Therefore $\hat{\mathbf{v}} \in V^F$.

(b) Define:

$$\mathcal{W}^F \equiv \{\mathbf{v} \in \Re^n \mid (\exists \mathbf{y} \in Y)(v_i = v_i^F(y_i), \quad \text{all } i)\}$$

The theorem claims that $\mathcal{V}^F = \mathcal{W}^F$.

Note that $\mathcal{V}^F \subseteq V^F = W^F$ and $\mathcal{W}^F \subseteq W^F = V^F$ from the definitions and part (a) of the theorem. Any member of $\mathcal{V}^F$ also satisfies (i)–(vi) above plus:

(vii) $\neg(\exists\ \tilde{\mathbf{v}} \in V^F)(\tilde{v}_i \geq v_i, \text{ all } i, \text{ and } \tilde{v}_i > v_i \text{ for some } i)$

Suppose there is a vector $\hat{\mathbf{v}}$ such that $\hat{\mathbf{v}} \in \mathcal{V}^F$ and $\hat{\mathbf{v}} \notin \mathcal{W}^F$. Since $\hat{\mathbf{v}} \in W^F$ there exists $\hat{\mathbf{y}} \in Y$ such that $v_i^F(\hat{y}_i) \geq \hat{v}_i$ for all i. Since $\hat{\mathbf{v}} \notin \mathcal{W}^F$ we know that for every $\mathbf{y} \in Y$ there is a region i' such that either $v_{i'}^F(y_{i'}) > \hat{v}_{i'}$ or $v_{i'}^F(y_{i'}) < \hat{v}_{i'}$. For $\hat{\mathbf{y}}$ in particular we cannot have the latter strict inequality for any i', since that would contradict the weak inequality just established, so there must be a region i' such that $v_{i'}^F(\hat{y}_{i'}) > \hat{y}_{i'}$.

Now construct the vector $(v_1^F(\hat{y}_1), \ldots, v_n^F(\hat{y}_n))$. This is a member of W^F since every component is positive, $\hat{\mathbf{y}} \in Y$, and the required inequality holds trivially. Therefore it also a member of V^F. However, $(v_1^F(\hat{y}_1), \ldots, v_n^F(\hat{y}_n)) \in V^F$, $v_i^F(\hat{y}_i) \geq \hat{v}_i$ for all i and $v_{i'}^F(\hat{y}_{i'}) > \hat{v}_{i'}$ imply $\hat{\mathbf{v}} \notin \mathcal{V}^F$, by (vii). This is a contradiction.

Conversely, suppose $\hat{\mathbf{v}} \in \mathcal{W}^F$ and $\hat{\mathbf{v}} \notin \mathcal{V}^F$. Since $\hat{\mathbf{v}} \in V^F$ we know there is a vector $\tilde{\mathbf{v}} \in V^F$ such that $\tilde{v}_i \geq \hat{v}_i$ for all i and $\tilde{v}_{i'} > \hat{v}_{i'}$ for some i'. We have $\hat{\mathbf{v}} \in \mathcal{W}^F$ while $\tilde{\mathbf{v}} \in W^F$, so there exists a vector $\hat{\mathbf{y}} \in Y$ and a vector $\tilde{\mathbf{y}} \in Y$ such that $v_i^F(\hat{y}_i) = \hat{v}_i$ for all i and $v_i^F(\tilde{y}_i) \geq \tilde{v}_i$ for all i. Putting these pieces together:

$$v_i^F(\tilde{y}_i) \geq \tilde{v}_i \geq \hat{v}_i = v_i^F(\hat{y}_i), \quad i = 1, \ldots, n$$

and at i':

$$v_{i'}^F(\tilde{y}_{i'}) \geq \tilde{v}_{i'} > \hat{v}_{i'} = v_{i'}^F(\hat{y}_{i'})$$

Using the explicit formula for $v_i^F(y_i)$ (recall Lemma 2), clearing both sides, summing over all i and using the definition of Y gives $\alpha l^\beta \tau^{1-\beta} = \sum_{i=1}^n \tilde{y}_i > \sum_{i=1}^n \hat{y}_i = \alpha l^\beta \tau^{1-\beta}$, a contradiction. □

Proof of Lemma (used to prove Theorem 3): This follows most easily from a famous result for sums of geometric means known as Hölder's inequality. We give a slight variation of the result as stated in Hardy, Littlewood and Polya (1934; Section 2.8, #13). □

Hölder's Inequality

Consider two sets of (strictly) positive real numbers $\{a_1, \ldots, a_n\}$ and $\{b_1, \ldots, b_n\}$. The sets are *proportional* if $a_i/b_i = a_j/b_j$ for all i and j. Consider two real numbers k and k' with $k \neq 1$. The two numbers are *conjugate* if $k' = \frac{k}{k-1}$.

Assume we are given two sets of positive real numbers and two real numbers k and k' with $k \neq 1$, $k \neq 0$, and k' conjugate to k. Then:

(i). If $k < 1$, then:

$$\sum_{i=1}^{n} a_i b_i > \left(\sum_{i=1}^{n} a_i^k\right)^{\frac{1}{k}} \left(\sum_{i=1}^{n} b_i^{k'}\right)^{\frac{1}{k'}}$$

unless $\{a_1^k, \ldots, a_n^k\}$ and $\{b_1^{k'}, \ldots, b_n^{k'}\}$ are proportional [in which case the two sides are equal].

(ii). If $k > 1$, then:

$$\sum_{i=1}^{n} a_i b_i < \left(\sum_{i=1}^{n} a_i^k\right)^{\frac{1}{k}} \left(\sum_{i=1}^{n} b_i^{k'}\right)^{\frac{1}{k'}}$$

unless $\{a_1^k, \ldots, a_n^k\}$ and $\{b_1^{k'}, \ldots, b_n^{k'}\}$ are proportional [in which case the two sides are equal].

Define:

$$a_i \equiv t_i^{\frac{1}{\alpha}}, \quad b_i \equiv (l/n)^{\frac{\gamma}{\alpha}}$$
$$k \equiv \frac{\alpha}{1-\beta}, \quad k' \equiv \frac{\alpha}{\gamma}$$

Clearly a_i and b_i are positive for all i; $k \neq 1$ and $k \neq 0$; k and k' are conjugate; and $\gamma < 0$ is equivalent to $k < 1$. We also have:

$$\begin{aligned}
\sum_{i=1}^{n} a_i b_i &= \sum_{i=1}^{n} t_i^{\frac{1}{\alpha}} (l/n)^{\frac{\gamma}{\alpha}} \\
&= (l/n)^{\frac{\gamma}{\alpha}} \sum_{i=1}^{n} t_i^{\frac{1}{\alpha}} \\
\left(\sum_{i=1}^{n} a_i^k\right)^{\frac{1}{k}} \left(\sum_{i=1}^{n} b_i^{k'}\right)^{\frac{1}{k'}} &= \left[\sum_{i=1}^{n} \left(t_i^{\frac{1}{\alpha}}\right)^{\frac{\alpha}{1-\beta}}\right]^{\frac{1-\beta}{\alpha}} \left[\sum_{i=1}^{n} \left((l/n)^{\frac{\gamma}{\alpha}}\right)^{\frac{\alpha}{\gamma}}\right]^{\frac{\gamma}{\alpha}} \\
&= l^{\frac{\gamma}{\alpha}} \left(\sum_{i=1}^{n} t_i^{\frac{1}{1-\beta}}\right)^{\frac{1-\beta}{\alpha}} \\
&= l^{\frac{\gamma}{\alpha}} \tau^{\frac{1-\beta}{\alpha}}
\end{aligned}$$

We can use the first inequality unless the two sets of numbers satisfy proportionality. The sets of numbers are proportional if and only if $a_i^k/b_i^{k'} = a_j^k/b_j^{k'}$ for all i and j. Since $b_i = b_j = l/n$, this reduces to the requirement that $t_i = t_j$ for all i and j. This cannot occur, by our stipulation that $\mathbf{t} \in \Re^n_{++} \setminus \mathbf{I}$. Thus, we obtain $(l/n)^{\frac{\gamma}{\alpha}} \sum_{i=1}^{n} t_i^{\frac{1}{\alpha}} > l^{\frac{\gamma}{\alpha}} \tau^{\frac{1-\beta}{\alpha}}$, which is equivalent to (A1). Equation (A2) follows mutatis mutandis.

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Notes

1. Garrett (1998) writes, "Market integration is thought to affect national policy autonomy through three basic mechanisms. These are trade competitiveness pressures, the multinationalization of production, and the integration of financial markets" (p. 791). Ohmae (1995), Garrett (1998) and Rodrik (2000) provide broad analyses of these trends, the likelihood they will continue and their implications for sovereignty.
2. This brief development of the normative model omits certain minor technicalities. It is straightforward to define allocations, feasibility, and region of residence along the fully technical lines established by Bewley (1981).
3. If there were r owners in each region then each group would have $r + (l/n)$ members.
4. For another example of this approach in a similar context, see Leite-Monteiro (1997).
5. Boadway and Bruce (1989), p. 141.
6. More generally, if each region contains r immobile agents, then v_i is a monotone transformation of both of these group welfare functions with $\alpha = r/[r + (l/n)]$. Thus, α is still small provided the number of owners in each region is small relative to the number of native workers.
7. These definitions are standard (See Mas-Colell, Whinston and Green, 1995, chapter 16). We do, however, impose the additional requirement that welfare is positive for all groups. This maintains consistency between the set of welfare possibilities and the set of feasible allocations, which gives every region some workers and every agent some income.
8. Formal proofs are in the supplemental appendix.
9. This is all standard; see Wildasin (1991, 1994).
10. The fact that the distribution of resources to workers is invariant to the distribution of income to owners is not quite as specialized as it appears. Bergstrom and Cornes (1983) establish necessary and sufficient conditions for there to exist efficient allocations in which the quantity of a public good is independent of the distribution of private goods across individuals. The welfare maximization problem under federalism is formally the same as the problem they consider, since the common level of worker consumption is formally analogous to a public good.

    Under the assumption (plus others) that (i) differences in public goods are always compensatable by private goods, then a necessary condition for this independence is (ii) preferences have a generalized quasi-linear form. In our case, independence holds even though $v_i(y_i, c^F)$ fails (ii). This does not violate their necessity result since v_i also fails (i).
11. The definition alone does not imply that given any pair of regimes that one dominates the other, but it does imply that if regime j dominates regime k then regime k cannot dominate regime j. If both occur then there exists $\mathbf{v}^k \in \mathcal{V}^k$ and $\mathbf{v}^j \in \mathcal{V}^j$ such that $v_i^k \geq v_i^j$ for all groups i, but we must also have $v_i^j > v_i^k$ for some i, a contradiction. Two frontiers could satisfy (a) by spanning different parts of the welfare space, but then they would violate (b).
12. The parameters in Figures 1 and 2 are $n = 2$, $l = 6$, $a_1 = a_2 = 1$, $t_1 = 100$, $t_2 = 175$, $\beta = .7$, and $\alpha = 1/(1 + l/n)) = .25$, so $\gamma = -.05$. The welfare levels are $(v_1^A, v_2^A) = (53.9, 94.4)$ and $(v_1^F, v_2^F) = (52.0, 93.0)$.
13. These are $y_1 + 3c_1 = 100(3)^{.7}$ and $y_2 + 3c_2 = 175(3)^{.7}$.
14. These are $y_1 + 3c = y_1^F + 3c^F$ and $y_2 + 3c = y_2^F + 3c^F$, where $c^F = 80.06$ is defined in Theorem 2 and $y_i^F = [V_i^F/(c^F)^{1-\alpha}]^{(1/\alpha)}$, so $y_1^F = 14.25$ and $y_2^F = 145.87$.
15. The parameters in Figures 3 and 4 are the same as in Figures 1 and 2 except $\beta = .85$, so $\gamma = .10$, and the welfare levels are $(v_1^A, v_2^A) = (63.6, 111.3)$ and $(v_1^F, v_2^F) = (70.0, 116.1)$.

16. The sold lines are $y_1 + 3c_1 = 100(3)^{.85}$ and $y_2 + 3c_2 = 175(3)^{.85}$. The dashed lines use $c^F = 100.67$, $y_1^F = 23.53$, and $y_2^F = 177.82$. These appear flatter than the corresponding lines in Figure 2 because we scale the vertical axes differently.
17. "Thus, free migration cannot lead to Pareto-improvements, and may lead to Pareto-inferior outcomes if, in the no-migration situation, owners of immobile factors are being taxed to provide transfer payments to mobile workers" (p. 650).
18. It might seem that Wildasin's social welfare function is a "regional" social welfare function. The function depends only on the number of workers initially located in the region, however. It is not clear in what sense it could represent the welfare of the region after the population changes through migration. Bossert (1990) presents axioms that justify letting average regional income represent regional social welfare even as the population changes. To compute this average, however, our parameter α would have to depend on the number of workers in a region after migration. The welfare functions in this paper are therefore not in the class considered by Bossert.
19. See Wildasin (1991) for further details. Note that the central government has some freedom in choosing the T_i to manipulate the equilibrium distribution of group welfare.
20. Obviously, the federal government would need a great deal of information to implement these policies. Our justification for setting this issue aside is that we are primarily interested in understanding how autarky can dominate federalism. This analysis is most compelling when the case for federalism is strong, i.e. – when the constraints on federal policy are few. Raff and Wilson (1997) consider the design of interregional transfers when the central government has incomplete information about the productivity of regional governments. In this case there are both adverse selection and moral hazard problems, since high productivity regions may benefit from acting as if they have low productivity and the grants themselves may inhibit productivity. Raff and Wilson show that the optimal grant function can lead to some important differences from the complete information case. In particular, it may be optimal for production to be inefficient.
21. Wildasin (1994), p. 639.
22. Of course, group membership is not really immutable, so group welfare should not be defined by consumption by the initial members over the very long run. For a related discussion, see Michel, Pestieau, Vidal (1998).

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