ECON 445
Spring 2007
Professor Paul Rothstein

Problem Set 1
Due February 6 at the start of class
Worth 32 points (2 points per question)
Show all of your work!

Problem 1 (2 points)

Consider a firm which plans to maximize its profit. It faces the following (inverse) demand function:

$$P(Q) = \frac{100 - Q}{2}$$

Its total cost function is:

$$TC(Q) = 2Q^2$$

1. What is its profit maximizing quantity of output, and for what price does it sell the output?

Problem 2 (10 points)

Suppose that you derive utility from consuming two commodities X and Y. Your utility function is:

$$U(X, Y) = X^{0.5} + Y$$

Suppose the price of X is 1 dollar, the price of Y is 4 dollars, and your income is 24 dollars. So your budget constraint is:

$$X + 4Y = 24$$

Assume you need to allocate your income to maximize your utility.

2. Write down the LaGrangean and the three first order conditions. Denote the Lagrange multiplier for the budget constraint as λ.

3. Solve explicitly (meaning, in this case, numbers) for the solution to the first order conditions: X^*, Y^*, and λ^*.

4. Compute the value of the objective function at the solution.

5. Suppose your income goes up to 36 dollars. Repeat the above 3 steps and solve for the new values of X^*, Y^*, and λ^*. What is the value of your utility now (i.e., at the new solution)?

6. Compare the optimal value of X^* in question 3 and 4. Carefully (but without doing any new computations) draw the budget constraints and indifference curves that illustrate this result.

Problem 3 (8 points)
A popular utility function called *Cobb-Douglas* takes the following form:

$$U(X, Y) = X^{\alpha} Y^{1-\alpha}, \quad 0 < \alpha < 1$$

A consumer with this utility function will maximize his utility subject to he budget constraint:

$$I = P_x X + P_y Y$$

7. Write down the Lagrangian and the three first order condition.

8. Calculate the optimal choice of X and Y.

9. Now apply implicit function theorem to compute the slope of the indifference curve passing through the point (X, Y), that is, the slope of the slice of $U(X, Y)$.

   Hint: Refer to lecture 2 on the SmartBoard.

10. The slope you computed above is the MRS_{xy}. At the optimal choice of X and Y, we have:

$$|MRS_{xy}| = \frac{P_x}{P_y}$$

   Verify this by using the X and Y you derived in question 8 in the formula for the MRS_{xy} you derived in question 9.

Problem 4 (6 points)
Suppose a firm uses the following technology in production:

$$Y = F(K, L) = K^{\frac{1}{3}} L^{\frac{1}{3}}$$

Assume that the price for Y is 27 dollars per unit, the price for K is 1 dollar per unit, and the price for L is 9 dollars per unit.

11. A technology has *Decreasing Return to Scale* if:

    $$F(tK, tL) < tF(K, L), \;\; forany\ t > 1.$$

    Does the technology $F(K, L) = K^{\frac{1}{3}} L^{\frac{1}{3}}$ have *Decreasing Return to Scale*?

12. Assume that in the short run, the firm's use of capital is fixed at $K = 64$. With this condition, solve the firm's profit maximization problem and compute the level of profit at the solution.

13. In the long run, the firm can vary its use of both K and L. Under this condition, solve the firm's profit maximization problem and compute the level of profit at the solution.

    Compare this profit level with the previous one. What do you find? Explain the reason behind this difference.

Problem 5 (6 points)
Suppose we have two firms each producing a single good with just capital as an input, using the following technologies:

$$f^1(k_1) = 16(2^{1/2})(k_1^{1/2})$$

$$f^2(k_2) = 8(2^{1/2})(k_2^{1/2})$$

Firm 1 sells output at price p_1 and firm two at price p_2. Each unit of capital costs r.

14. Write down the profit maximization problem of firm 1 and solve for its demand for capital.

15. Write down the profit maximization problem of firm 2 and solve for its demand for capital.

16. Suppose $p_1 = p_2 = 2$ and there are exactly 10 units of capital available. What must r be so that the supply of capital exactly equals demand (so the market for capital clears)?