Lecture 7

Outline

1. Overview
2. The "indirect compensation function" (or "money metric indirect utility") and the equivalent variation
3. Excess burden of commodity taxes (the "standard" measure)

1. Overview

   (a) "The deadweight loss [or excess burden] from a tax system is that amount that is lost in excess of what the government collects." (Auerbach)

   (b) The idea that taxes have welfare consequences that are not fully captured by the revenue gathered is very old. However, before the nineteenth century, the extra costs that people had in mind came from administration, penalties and having to prove the extent of one's liability.

   (c) The notion of an excess burden caused by changing relative prices is relatively new. Dupuit (1844) is credited with basically getting it right.[1] However, the earliest widely studied discussion of the issue is in Marshall.

   (d) The phenomenon we are trying the measure is nicely described by Rosen in his public finance text.

   Suppose the government places a tax on ice cream. Consider someone who likes ice cream and consumes some before taxes but now, because of the tax, consumes zero.

   This person is obviously worse off. While he pays no money to the government, he is reallocating his income and buying a bundle that he could have afforded earlier but chose not to.

   On the other hand, since he pays no money to the government, his nominal tax burden is zero. All of this welfare loss is "excess burden."

   (e) People (non-economists) sometimes ask whether this is a "real" issue.

   Well, is there any basis for calling some welfare losses "real" and others "not real"? Could the source of the loss provide this basis?

   Economics does not make these distinctions. If people are worse off, by their own reckoning, then they are worse off, period.

[1] Originally in French. An English translation appears as, "On the Measurement of the Utility of Public Works," in *Readings in Welfare Economics*, Arrow and Scitovsky, 1969. Dupuit also discusses the phenomenon described by the Laffer curve, and he calls for more use of mathematics, "despite the anathema which economists of all times have pronounced against the latter."

(f) Before turning to technical matters, we should also consider how the concept is used.

   i. In most of economic analysis, efficiency is a binary concept. An allocation is either efficient or it is not.

   ii. Excess burden is a way of quantifying inefficiency.
   We want this because in policy analysis (tax policy as well as other policy) we want to compare the relative merits of allocations (created by different policies) all of which are inefficient. The degree of inefficiency is relevant to this comparison.

   iii. It would be very useful to have a book or a survey article that compiles all of the estimates and conventional wisdom about which taxes tend to produce the biggest excess burdens. It would provide the context necessary for declaring whether a particular excess burden is "big" or "small," and it would facilitate discussion of alternative sources of revenue.
   Maybe such a thing already exists?

   iv. The finding of a big excess burden does not *in itself* mean that a tax is "bad," because other aspects, like the equity of the tax burden, are also relevant in evaluating the tax.
   If there are two possible tax instruments and the higher excess burden is associated with the more equitable tax, then there is an equity/efficiency tradeoff.
   Calling attention to THIS tradeoff is ALSO part of economics, even when there is little possibility of quantifying it.

(g) What do we want from a measure of excess burden? Here are some common *desiderata* in the literature.

   The only paper I know along these lines is, Auerbach and Rosen, "Will the Real Excess Burden Please Stand Up? (Or, seven measures in search of a concept)," in *The Fiscal Behavior of State and Local Governments, Selected Papers of Harvey S. Rosen*, Edward Elgar, 1997.

   i. It should be unaffected by monotonic transformations of utility functions.

   ii. It should be zero under lump-sum taxes, strictly positive otherwise.

   iii. In a single-consumer economy, if two taxes raise the same revenue, then the tax with the higher excess burden should make the consumer worse off than the tax with the lower excess burden.
   Note that this assures that optimal taxes defined by utility maximization also minimize deadweight loss.

   iv. The dollar value of the excess burden should have a clear interpretation, at least for a single-consumer economy.

v. The tax revenue raised shouldn't just disappear.

(h) What we are after.

Note! For purposes of this picture only, good 0 is the taxed good and good 1 is the numeraire. This is only so we can identify distance on the vertical axis with money. The formal analysis below does not require this. When we return to the optimal tax problem, we will again want good 0 to be both untaxed and numeraire.

Figures 1, 2, 3

2. The "indirect compensation function" (or "money metric indirect utility") and the equivalent variation

The terminology is from Varian (1984). The ideas follow Pauwels (1986) fairly closely, with some additions from Mas-Colell, Whinston and Green (1995).

The notation tries to follow the notation we have developed for the optimal commodity tax model.

(a) $n+1$ commodities

$x \in R^{n+1}$

$q \in R^{n+1}$

$U(x)$, utility

$qx = I$, budget constraint

(b) Indirect utility

Given a price vector and income pair (q, I):

$$V(q, I) = \max\ U(x)\ \text{ subject to }\ qx = I$$

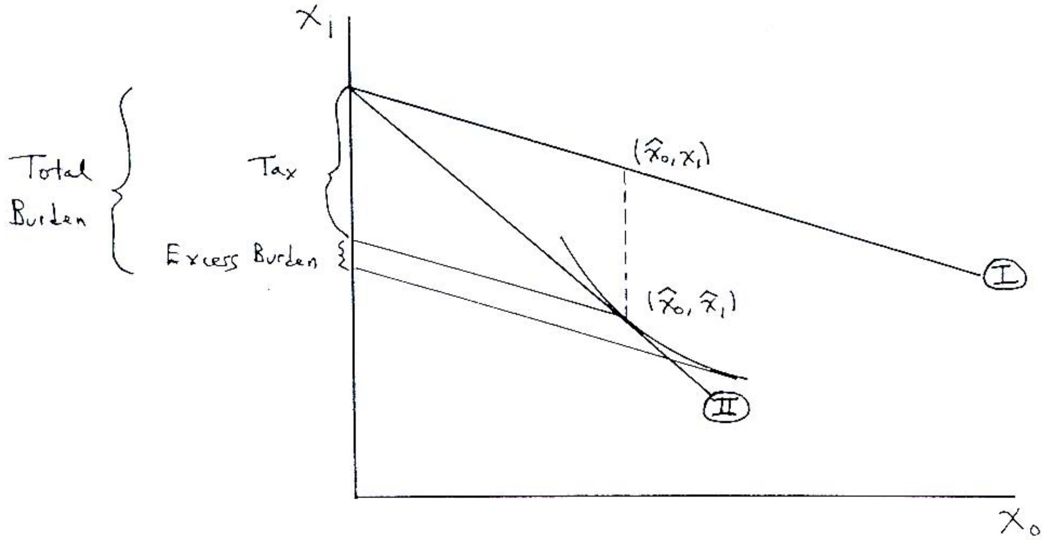
(c) Money metric indirect utility

Given q, q' and I:

$$E[q, V(q', I)] = \min\ qx\ \text{ subject to }\ U(x) = V(q', I)$$

An important property of money metric indirect utility is the identity:

$$E[q, V(q, I)] = I \tag{1}$$



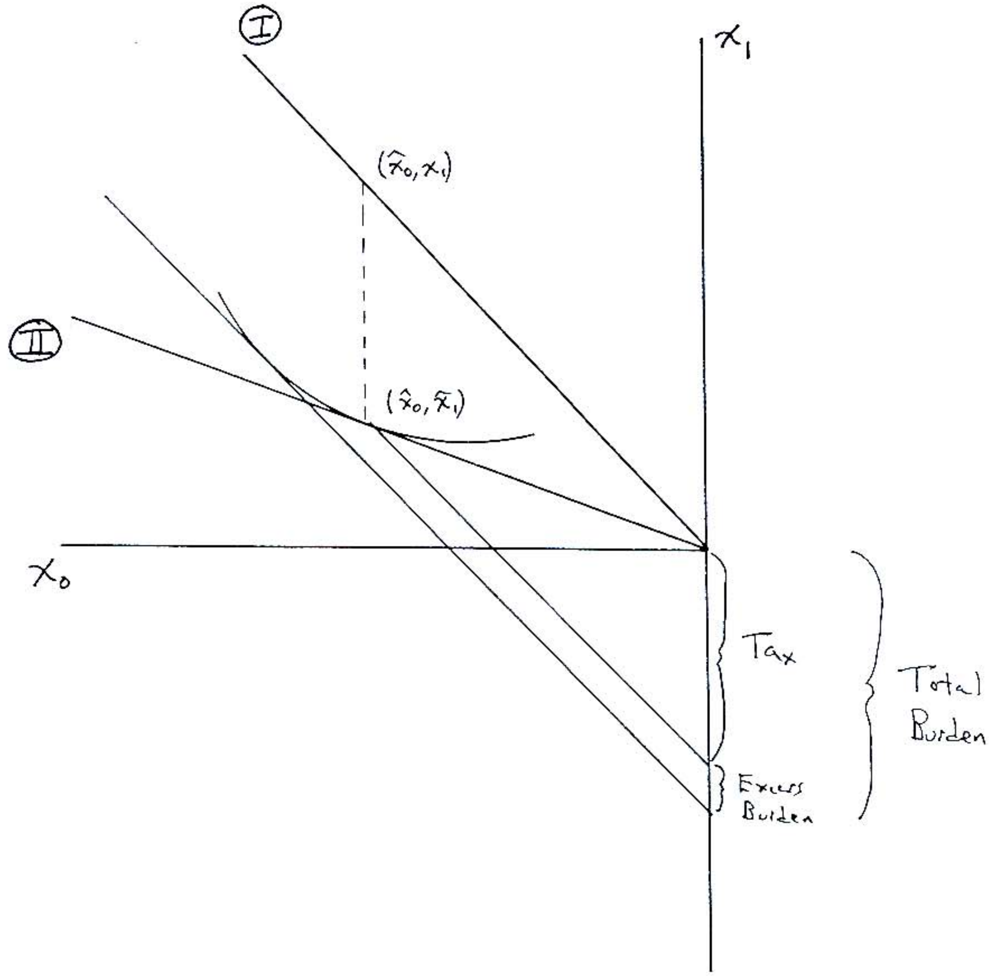
Constraint (I) is $q_0 x_0 + x_1 = I$

Constraint (II) is $(q_0 + t_0) \cdot x_0 + x_1 = I$

$\boxed{\text{Tax} = x_1 - \hat{x}_1}$ because

$$x_1 - \hat{x}_1 = I - q_0 \hat{x}_0 - [I - (q_0 + t_0)\hat{x}_0]$$
$$= t_0 \cdot \hat{x}_0$$

Figure 1



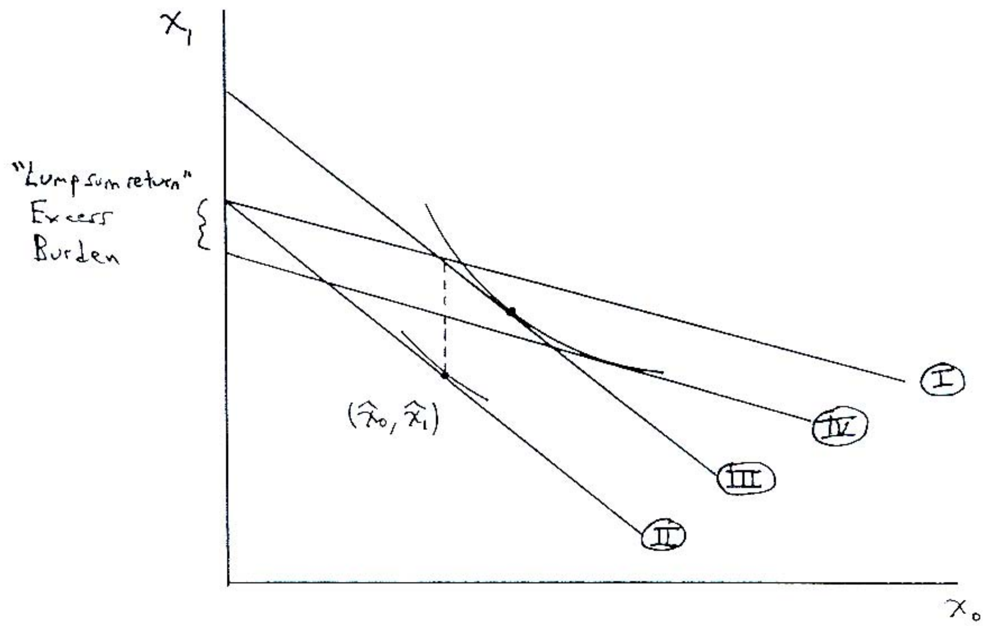
Constraint (I) is $q_0 x_0 + x_1 = 0$

Constraint (II) is $(q_0 + t_0) x_0 + x_1 = 0$

q_0 is the wage, x_0 is (negative) labor supply

$$x_1 - \hat{x}_1 = -q_0 \hat{x}_0 - [-(q_0 + t_0)\hat{x}_0] = t_0 \hat{x}_0$$

Figure 2



The indifference curve comes from maximizing utility subject to (III). This constraint is defined by post-tax prices and income after return of the tax revenue.

Figure 3

(d) Equivalent variation

Given the following initial conditions or "states":

$$o = (q^o, I^o)$$
$$i = (q^i, I^i)$$

The equivalent variation for the transition from o to i solves:

$$V(q^o, I^o + EV^{oi}) = V(q^i, I^i)$$

That is to say:

> *EV^{oi} is the money you must be given in state o to be as well off as you are in state i. If you prefer state i to state o, it is the (smallest) amount you would be willing to accept in state o to forego state i. If you prefer state o to state i, it is the (negative of the largest) amount you would be willing to pay in state o to forego state i.*

From the definition and (1) we have:

$$E[q^o, V(q^i, I^i)] = E[q^o, V(q^o, I^o + EV^{oi})] = I^o + EV^{oi}$$

Rearranging then gives:

$$EV^{oi} = E[q^o, V(q^i, I^i)] - I^o \tag{2}$$

We can also replace I^o using (1):

$$EV^{oi} = E[q^o, V(q^i, I^i)] - E[q^o, V(q^o, I^o)]$$

Note two points:

i. From (2), it is clear that *only one indifference curve is needed* to construct the equivalent variation.
It is common to draw the indifference curve in the initial state, but it is actually superfluous.

ii. From the expression after (2), it is clear that *the reference price vector remains fixed*. The value of this becomes clear below.

(e) Key properties of the equivalent variation:

i. Given two states $o, 1$, the equivalent variation for the transition from o to 1 is positive if and only if the consumer prefers 1 to o:

$$\begin{aligned} EV^{o1} > 0 &\iff E[q^o, V(q^1, I^1)] - E[q^o, V(q^o, I^o)] > 0 \\ &\iff E[q^o, V(q^1, I^1)] > E[q^o, V(q^o, I^o)] \\ &\iff V(q^1, I^1) > V(q^o, I^o) \end{aligned}$$

ii. Given three states $o, 1, 2$ the equivalent variation for the transition from o to 1 is greater than the equivalent variation for the transition from o to 2 if and only if the consumer prefers 1 to 2:

$$\begin{aligned} EV^{o1} > EV^{o2} &\iff E[q^o, V(q^1, I^1)] - I^o > E[q^o, V(q^2, I^2)] - I^o \\ &\iff E[q^o, V(q^1, I^1)] > E[q^o, V(q^2, I^2)] \\ &\iff V(q^1, I^1) > V(q^2, I^2) \end{aligned}$$

3. Excess burden of commodity taxes (the "standard" measure)

(a) The analysis of excess burden here assumes a single-consumer economy and constant producer prices (linear technology).

The results generalize fairly well to more general technology (Auerbach (1985), section 3).

In the many-consumer economy, it is not possible to define aggregate measures of excess burden that are independent of the initial distribution of income, except under conditions of "exact aggregation" (Auerbach (1985), section 3).

(b) State o is the initial untaxed state. $\texttt{D}$ is a state with commodity (or *distorting*) tax vector t that raises revenue $tx(q^o + t, I^o)$. $\texttt{L}$ is a state with a lump-sum tax equal to the revenue raised by t.

$$\begin{aligned} &\text{state } o \text{ (no taxes)}: && \begin{matrix} q^o \\ I^o \end{matrix} \\ &\text{state } \texttt{L} \text{ (lump-sum taxes)}: && \begin{aligned} q^{\texttt{L}} &= q^o \\ I^{\texttt{L}} &= I^o - tx(q^o + t, I^o) \end{aligned} \\ &\text{state } \texttt{D} \text{ (distorting taxes)}: && \begin{aligned} q^{\texttt{D}} &= q^o + t \\ I^{\texttt{D}} &= I^o \end{aligned} \end{aligned}$$

(c) We now define:

$$\text{Excess Burden} \equiv -EV^{\texttt{LD}} = I^{\texttt{L}} - E[q^{\texttt{L}}, V(q^{\texttt{D}}, I^{\texttt{D}})]$$

That is to say:

> *The excess burden of t is a positive number denoting the amount of money you must forego in state* $\mathtt{L}$ *(the equivalent lump-sum tax state) to be as badly off as you are in state* $\mathtt{D}$ *(the commodity tax state). Or, put more intuitively, it is the most you are willing to pay for the opportunity to pay your taxes lump-sum instead of through distorting taxes.*

Figure 1A

(d) This measure will satisfy all of our axioms except the fifth. The problem is that the tax revenue just disappears.

i. Property 1:
Equivalent variations are unaffected by monotonic transformations of utility.

ii. Property 2:
Clear.

iii. Property 3:
Consider two commodity tax vectors, $t^{\mathtt{D}}$ and $t^{\mathtt{D}'}$, that each raise revenue T. The excess burden of $t^{\mathtt{D}}$ exceeds the excess burden of $t^{\mathtt{D}'}$ if and only if:

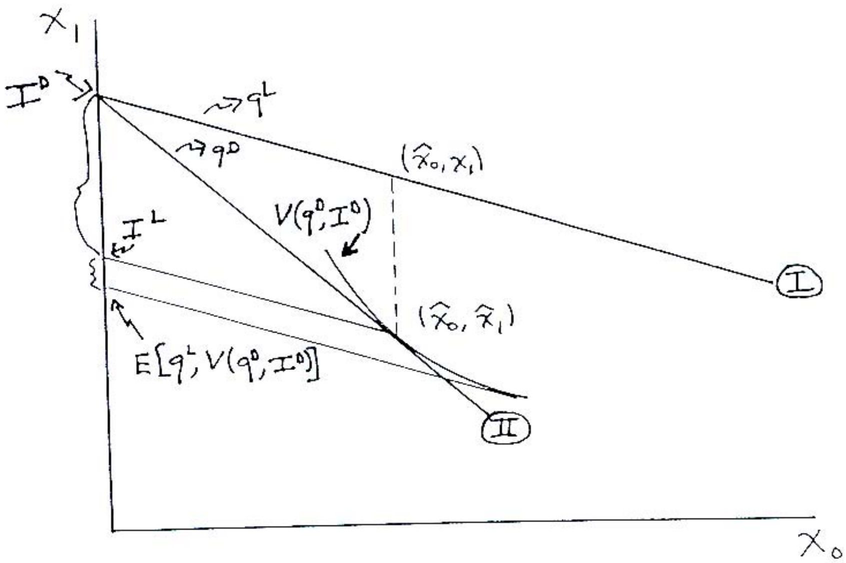
$$-EV^{\mathtt{LD}} > -EV^{\mathtt{LD}'}$$

$$\begin{aligned}
&\iff I^{\mathtt{L}} - E[q^{\mathtt{L}}, V(q^{\mathtt{D}}, I^{\mathtt{D}})] > I^{\mathtt{L}} - E[q^{\mathtt{L}}, V(q^{\mathtt{D}'}, I^{\mathtt{D}'})] \\
&\iff E[q^{\mathtt{L}}, V(q^{\mathtt{D}'}, I^{\mathtt{D}'})] > E[q^{\mathtt{L}}, V(q^{\mathtt{D}}, I^{\mathtt{D}})] \\
&\iff V(q^{\mathtt{D}'}, I^{\mathtt{D}'}) > V(q^{\mathtt{D}}, I^{\mathtt{D}})
\end{aligned}$$

The tax generating the higher excess burden leads to a state with lower utility.

iv. Property 4 is also satisfied by the fact that the excess burden is itself an equivalent variation.
However, the standard interpretation comes with a little re-writing.

$$\begin{aligned}
-EV^{\mathtt{LD}} &= -[E(q^{\mathtt{L}}, V(q^{\mathtt{D}}, I^{\mathtt{D}})) - I^{\mathtt{L}}] \\
&= -[E(q^{\mathtt{L}}, V(q^{\mathtt{D}}, I^{\mathtt{D}})) - (I^o - tx(q^o + t, I^o))] \\
&= -[E(q^o, V(q^{\mathtt{D}}, I^{\mathtt{D}})) - (I^o - tx(q^o + t, I^o))] \\
&= I^o - E[q^o, V(q^{\mathtt{D}}, I^{\mathtt{D}})] - tx(q^o + t, I^o) \qquad (3) \\
&= -EV^{oD} - tx(q^o + t, I^o) \qquad (4)
\end{aligned}$$



$$\text{Excess Burden} = -EV^{L0} = I^L - E[q^L, V(q^0, I^0)]$$

Figure 1A

where we use the fact $I^{\mathtt{L}} = I^o - tx(q^o + t, I^o)$ and $q^{\mathtt{L}} = q^o$.
$-EV^{oD}$ can be interpreted as the "total burden" of the commodity tax. Equation (4) therefore says that the excess burden equals the total burden less the tax revenue raised.

Figure 1B

(e) There is a small but dreadful literature on whether excess burden "should" be defined by subtracting the revenue actually raised by the commodity taxes (as in (3)) or the revenue that would be raised by those taxes in some compensated equilibrium.

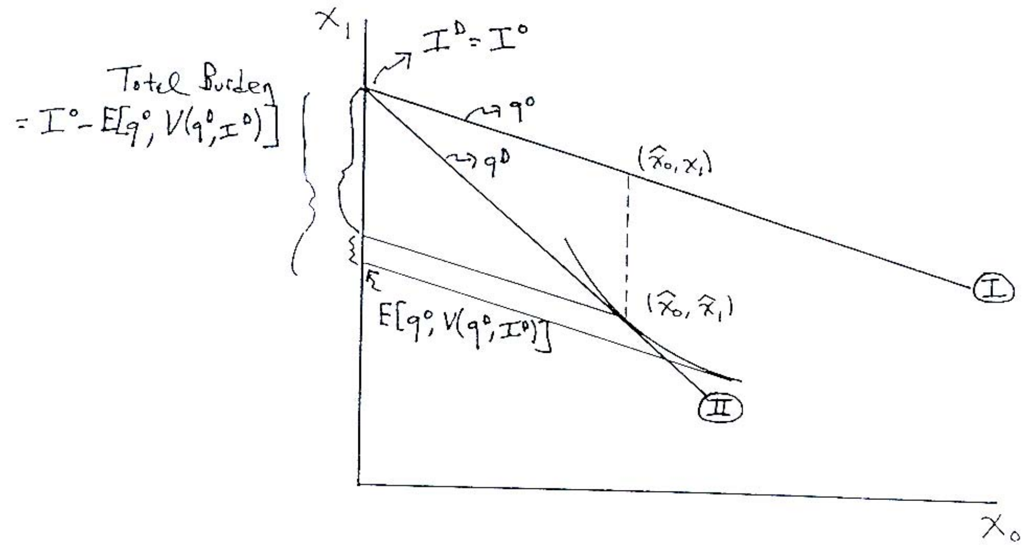
The measure of excess burden in the literature that subtracts the revenue raised in the compensated equilibrium *and* satisfies axiom 4 (it is derived from the compensating variation) violates axiom 3.

For more on this, see the papers by Kay (1980) and Pauwels.

We have avoided this discussion entirely! We developed our measure of excess burden from first principles.

(f) We use equation (3) to define "total excess burden" since all dependence on t is explicit:

$$\mathrm{TEB}^{\mathtt{LD}}(t) \;=\; I^o - E[q^o, V(q^o + t, I^o)] - tx(q^o + t, I^o)$$



$$\text{Excess Burden} = -EV^{LD}$$
$$= I^0 - E[q^0, V(q^D, I^D)] - T$$
$$= -EV^{OD} - T$$

Figure 1B