Lecture 6

Atkinson and Stern

Outline

1. On the "underprovision" and "overprovision" of public goods
2. Atkinson-Stern model
3. Comparing "rules" at the optimum
4. Comparing "levels" at the optimum

1. On the "underprovision" and "overprovision" of public goods

   (a) In a standard economy, the "first-best" Pareto problem yields three rules (first order conditions) that are necessary conditions for efficiency. With convexity these are also sufficient. See Mas-Colell et al., Chapter 16, expecially 16F.

   (b) If we extend the economy so it includes a public good, the Pareto problem yields the three previous rules for the private goods, the usual efficiency condition for the use of factors in producing the public good (the marginal rates of technical substitution must be the same as in the production of private goods), and a new rule.

   The new rule is called the "Samuelson condition."

   More formally, suppose there is one private good x and one public good e (so the rules for pairs of private goods will not appear) and H individuals. Fix an initial distribution of utility for individuals 2 through H, $\bar{U}^2, ..., \bar{U}^H$. Let F denote the transformation function (including the restrictions on aggregate resources that are part of feasibility).

   An efficient allocation must solve:

$$\begin{array}{cl} \underset{x^1,...,x^H,e}{\text{Max}} & U^1(x^1, e) \\ \text{s.t.} & U^2(x^2, e) = \bar{U}^2 \\ & \vdots \\ & U^H(x^H, e) = \bar{U}^H \\ & F\left(\sum_{h=1}^{H} x^h, e\right) = 0 \end{array}$$

   Denote the quantity of public good in the solution by

$$e_{LS}(.) \equiv e_{LS}(\bar{U}^2, ..., \bar{U}^H)$$

"LS" stands for "lump sum."

The quantity e_{LS} satisfies:

$$\text{MRT}(e_{LS}(.)) = \sum_{h=1}^{H} \text{MRS}^h(e_{LS}(.)) \tag{1}$$

This is the Samuelson condition.

i. As the initial distribution of utility changes, the values of all of the choice variables, including $e_{LS}(.)$, generally changes. The Samuelson condition holds at all of these vectors. It is a condition that must hold at all efficient allocations.

   Put a little differently, even if we fix the value of all of the other choice variables and the Samuelson condition determines a unique value of public good, this is not "the" efficient quantity of public good. It is just the quantity in one particular efficient allocation.

   It is in this sense that the Samuelson condition does not determine "the" efficient quantity of public good. But you should not expect it to, since in general there is no such thing as "the" efficient level of a public good.

ii. "In general" isn't "always," of course.

   The best analysis of restrictions on preferences that imply that there is a unique efficient quantity of public good is:

   Bergstrom, T.C. and R.C. Cornes, 1983, "Independence of allocative efficiency from distribution in the theory of public goods," *Econometrica* 51, 1753-1765.

   The restrictions generalize quasi-linearity, but they are still pretty strong.

(c) One possible benchmark for "the" efficient level of a public good comes from maximizing a social welfare function subject to the transformation function. This is the "pure planner problem."

Formally:

$$\begin{array}{cc} \underset{x^1, ..., x^H, e}{\text{Max}} & \sum_{h=1}^{H} U^h(x^h, e) \\ \text{s.t.} & F\left(\sum_{h=1}^{H} x^h, e\right) = 0 \end{array}$$

Whether or not this solution can be decentralized is the content of the second welfare theorem for this economy. The answer is generally "yes," via the Lindahl equilibrium *with an appropriate set of lump-sum transfers.*

i. The government uses taxes and transfers to make everyone's endowments equal to the bundles they should consume. The government then announces personalize prices and people optimize by consuming these exact bundles.

ii. Note that this is a rough description since there is nothing sequential about the Lindahl equilibrium. Everything takes place simultaneously, just as in the second welfare theorem for the standard economy.
    See Duncan Foley, "Lindahl's Solution and the Core of an Economy with Public Goods," *Econometrica*, 1970.

(d) Another possible benchmark for "the" efficient level of a public good is the amount that would exist in the Lindahl equilibrium for the economy *without the lump-sum transfers.*

In other words, consider the quantity of public good that emerges in a Lindahl equilibrium that respects the initial distribution of endowment. This is also efficient.

There is no reason to expect this amount to be the same as in the allocation solving the SWF maximization problem.

This is probably the right benchmark against which to compare equilibrium allocations with public goods that arise from the Cournot-Nash equilibrium in a voluntary contribution game, the Nash equilibrium in a prisoner's dilemma game (really a special case of the first), and voting equilibrium. The reason is that the government plays a minimal role in these equilibria and social welfare is completely absent. This suggests that the proper comparison is with an efficient allocation that respects the initial distribution of income, as does the Lindahl equilibrium without transfers.

i. The relationship between voting equilibrium and Lindahl equilibrium is nicely discussed in a famous paper by Bergstrom and Goodman, "Private Demands for Public Goods," *American Economic Review*, 1973.

It is always worth noting that the Lindahl equilibrium concept has problems.

There are lots of reasons people give.

i. One obvious one is the amount of information the government must have, in order to select the personalized prices.
ii. I think there is an even more basic problem. People must believe that the only amount of the public good they will receive is the amount they pay for. The personalized price for the good clears a personalized market. As far as each person is concerned this personalized equilibrium quantity is all they will consume. See Mas-Colell et al., Chapter 11C and also p. 569.
    This is not plausible if the good is nonexcludable. In fact, it is hardly plausible even if the good is excludable, because exclusion is always costly. It is true that no exclusion would actually occur in Lindahl equilibrium – exclusion would be inefficient – but the threat of (costly) exclusion would have to be credible.

iii. Both problems are addressed in the implementation literature. One solution is the "demand revelation mechanism.

   See Chapter 5 in Laffont, *Fundamentals of Public Economics*, for a brief but thorough discussion of this topic.

(e) Yet another benchmark for the efficient level of a public good is the "full optimum" allocation in Atkinson-Stern. This is an equilibrium allocation – it is an efficient equilibrium, like Lindahl equilibria.

   They write that under- or over- provision could mean different things.

   > There are a number of possible interpretations which could be given.... [I]t could mean that the optimum output levels where public goods are financed by distortionary taxation are larger or smaller than the levels in the full optimum (financed by lump sum taxation). The latter interpretation is perhaps the more interesting and it is the one on which we focus – see Section IV.

   The construction is very specific to their model. They have h *identical* individuals and they assume *fixed producer prices.* Individuals have maximized utility subject to a budget constraint $p^*x = -T$. Note that there are no commodity taxes in *this* problem. This gives indirect utility $V(T, e)$.

   The government then solves:

$$\begin{array}{ll} \underset{T,e}{\text{Max}} & hV(T,e) \\ \text{s.t.} & hT = e \end{array}$$

   Recall that fixed producer prices allow you to view e as either a quantity of public good or a fixed amount of revenue.

   The resulting equilibrium allocation will be efficient and satisfy the Samuelson condition.

   The appropriate generalization of this problem would be to permit a general SWF, individuals with different tastes and differentiated lump-sum taxes.

   The exact relationship between this outcome and Lindahl equilibrium requires some further thought.

2. Atkinson-Stern model (we follow their notation).

   (a) *Households*

   There are h identical households.

   We cannot really have "public goods" in a model with only one household.

   Preferences:

$$U(x, e)$$

Budget Constraint:

$$qx = 0$$

Assume the first element of the vector, x_1, is labor supply. Then q_1 is the wage. Make this the numeraire, so:

$$q_1 = 1$$

Utility Maximization:

$$\mathcal{L} = U(x, e) - \alpha q x$$

First order conditions:

$$\frac{\partial U}{\partial x_i} = \alpha q_i, \qquad i = 1, ..., n$$

This gives rise to the vector of individual factor supplies and demands:

$$x(q, e)$$

and indirect utility:

$$V(q, e)$$

Note that $\frac{\partial V}{\partial q_i} = -\alpha x_i$ (do not confuse this with the first order condition). Aggregate demands are:

$$X(q, e) = h x(q, e)$$

(b) *Production*

They suppose e is produced entirely by the private sector, bought by the government using tax revenue, and given to consumers. These are the same assumptions we made about x^G in the previous lectures. The only differences are that now e is endogenous and there are multiple individuals.

Production is constant returns to scale (they do not use the stronger assumption of linear technology).

Firms solve:

$$\max_y \; py \quad \text{s.t.} \quad G(y, e) = 0$$

$$\mathcal{L} = py - \gamma G(y, e)$$

Define $G_i \equiv \partial G / \partial y_i$. Then:

$$p_i = \gamma G_i, \qquad i = 1, ..., n$$

With good 1 untaxed, we have:

$$p_1 = 1$$

If we take the ratio with the first equation then we have the $n-1$ conditions:

$$p_i = \frac{G_i}{G_1}, \quad i = 2, ..., n$$

Recalling our analysis in a previous lecture, we assume $G_1 = 1$ at all vectors (y, e) without any real loss of generality. Thus:

$$p_i = G_i, \qquad i = 1, ..., n$$

(c) *Government*

Social Welfare Function:

$$\text{SWF} = hV(q, e)$$

(d) Modified Samuelson Condition at the Second-Best Optimum

The government solves:

$$\begin{array}{cc} \underset{q_2, ..., q_n, e}{\text{Max}} & hV(q, e) \\ \text{s.t.} & G[X(q, e), e] = 0 \end{array}$$

This gives:

$$\mathcal{L} = hV(q, e) - \lambda G[X(q, e), e]$$

Attachment

At the solution:

$$\text{MRT} = \frac{\alpha}{\lambda} \sum \text{MRS} + \frac{\partial}{\partial e}\left[\sum_{i=1}^{n} t_i X_i\right] \tag{2}$$

3. Comparing "rules" at the optimum

(a) Comparing (1) and (2) provides some intuition about the nature of the optimum.

Also, as part of the set of first order conditions, (1) and (2) play a role in deriving local comparative statics results. Atkinson-Stern consider how much public good would change at the first-best optimum with lump-sum taxes from a small shift to optimal distorting taxes.

(b) The right-most term is the revenue effect from the impact of the public good on taxes paid due to changes in consumption of private goods that are complements and substitutes to the public good.

Insofar as this term is positive, at the solution we would have:

$$\text{MRT} > \sum \text{MRS}$$

$$\mathcal{L} = h V(q,e) - \lambda G[X(q,e), e]$$

$$\frac{\partial \mathcal{L}}{\partial e} = h \frac{\partial V}{\partial e} - \lambda \left[G_1 \cdot \frac{\partial x_1}{\partial e} + \ldots + G_n \frac{\partial x_n}{\partial e} + G_e \right]$$

$$= 0$$

$$\Rightarrow h \cdot \partial V / \partial e = \lambda G_e + \lambda \cdot \sum_{i=1}^{n} G_i \frac{\partial x_i}{\partial e}$$

Divide by αq_k:

$$\Rightarrow h \cdot \frac{\partial V / \partial e}{\alpha q_k} = \frac{\lambda}{\alpha q_k} G_e + \frac{\lambda}{\alpha q_k} \sum_{i=1}^{n} G_i \frac{\partial x_i}{\partial e}$$

Use $G_i / p_i = 1$ on r.h.s.:

$$\Rightarrow h \cdot \frac{\partial V / \partial e}{\alpha q_k} = \frac{p_k}{q_k} \cdot \frac{\lambda}{\alpha} \cdot \frac{G_e}{G_k} + \frac{\lambda}{\alpha q_k} \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial e}$$

Multiply by $\frac{\alpha}{\lambda}$.

At $k=1$ we have $q_1 = p_1 = 1$ and $\alpha q_1 = \frac{\partial U}{\partial x_1}$.

$$\Rightarrow \frac{\alpha}{\lambda} \cdot h \cdot \frac{\partial V / \partial e}{\partial U / \partial x_1} = \frac{G_e}{G_1} + \sum_{i=1}^{n} p_i \cdot \frac{\partial x_i}{\partial e}$$

$$\Rightarrow \quad \frac{G_e}{G} = \frac{\alpha}{\lambda} \, h \cdot \frac{\partial U/\partial e}{\partial U/\partial x_1} - \sum_{i=1}^{n} (q_i - t_i) \frac{\partial X_i}{\partial e}$$

$$MRT = \frac{\alpha}{\lambda} \sum MRS + \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial e}$$

$$\boxed{MRT = \frac{\alpha}{\lambda} \sum MRS + \frac{\partial}{\partial e} \sum_{i=1}^{n} t_i X_i}$$

(Of course $t_1 = 0$ here).

If we interpret MRT as "direct costs" and $\sum MRS$ as "direct benefits," then this says that at the optimum a small increase in the amount of public good should generate direct costs that exceed direct benefits. This is an optimum when ancillary benefits are high.

(c) Now assume this term is zero to focus on the remaining effects:

$$\text{MRT} = \frac{\alpha}{\lambda} \sum \text{MRS}$$

The first order conditions in the consumer prices (q_i) give the same equations we used to derive the Ramsey Rule. Making the publicly provided good "endogenous," in the sense that the government chooses it, does not affect the *form* of the first order conditions for the tax rates.

Using those (recall earlier lectures) in the expression above (completely straightforward) gives:

$$\frac{\alpha}{\lambda} = 1 - \sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I} + \sum_{i=1}^{n} t_i (S_{ik}/X_k), \qquad k = 2, ..., n$$

(d) There is an intuition that, at the optimum (and having ruled out ancillary benefits of the kind discussed above), a small increase in the amount of public good should generate direct costs that are less than direct benefits. The reason is that the distortion from commodity taxes exists as an ancillary cost.

This leads one to expect $\alpha < \lambda$ and

$$\text{MRT} < \sum \text{MRS}$$

However, things are not quite that simple. There are two effects.

i. The term $\sum_{i=1}^{n} t_i (S_{ik}/X_k)$.
Given that the taxes are optimal and the revenue that must be raised is positive, this is negative (recall the analysis of θ in Lecture 3).
It works in the direction of $\alpha < \lambda$, so it tends to produce:

$$\text{MRT} < \sum \text{MRS}$$

ii. The term $\sum_{i=1}^{n} t_i \frac{\partial X_i}{\partial I}$.
The taxes themselves create price changes, which create income effects, which in turn create "revenue effects."
If all tax rates are positive and fall on normal goods then this term is positive. We could then be sure that $\alpha < \lambda$.
On the other hand, with leisure normal, labor hours decrease with income. In the notation of this paper labor supply is a negative number, so a decrease in labor hours means, say labor hours go from -12 to -10. Therefore:

$$\frac{\partial X_i}{\partial I} > 0$$

Thus, the product may be negative and nothing rules out that the sum is negative.

(e) Consider a couple of cases.

i. Suppose there is one commodity and leisure.
If the commodity is the taxed good and it is normal, then the tax rate on it must be positive. This gives $t_i \frac{\partial X_i}{\partial I} > 0$, therefore $\alpha < \lambda$ and $\text{MRT} < \sum \text{MRS}$.
Switching to a tax on leisure is without any loss of generality. Regardless of what may happen to some intermediate quantities, terms that depend only on the final allocation must remain the same. In particular we must still have $\text{MRT} < \sum \text{MRS}$. This is true whether leisure is normal or inferior!

ii. If there are $n-1$ commodities and leisure, we cannot say what will happen. Even if all the commodities are normal and taxed and leisure is untaxed, some of the tax rates may be negative and the sign of the income effects term is ambiguous.

4. Comparing "levels" at the optimum

(a) What can we say about e_{LS} versus e_{CT}, the optimum with commodity taxes?
This paper does not say much about this question.
They do show that under very specific conditions, a small reduction in lump-sum tax will cause a reduction in public good provision.
They also show that a technical rule from an earlier paper concerning this issue is not correct.

(b) We have:
$$\text{MRT}(e_{CT}) = \frac{\alpha}{\lambda} \sum \text{MRS}(e_{CT})$$
If $\alpha < \lambda$ then:
$$\text{MRT}(e_{CT}) < \sum \text{MRS}(e_{CT})$$
Let's interpret these as functions of just the public good – the first order conditions for the commodity taxes are incorporated and the taxes adjust continuously as the public good changes.
The Samuelson rule gives us:
$$\text{MRT}^*(e_{LS}) = \sum \text{MRS}^*(e_{LS})$$
where now the lump-sum tax adjusts continuously as the public good changes. We use asterisks to indicate that *these curves need not be the same as the earlier ones since the tax instruments are different.*

If $\sum$MRS and If $\sum$MRS* are similar and globally decreasing and MRT and MRT* are similar and globally increasing, then we can conclude:

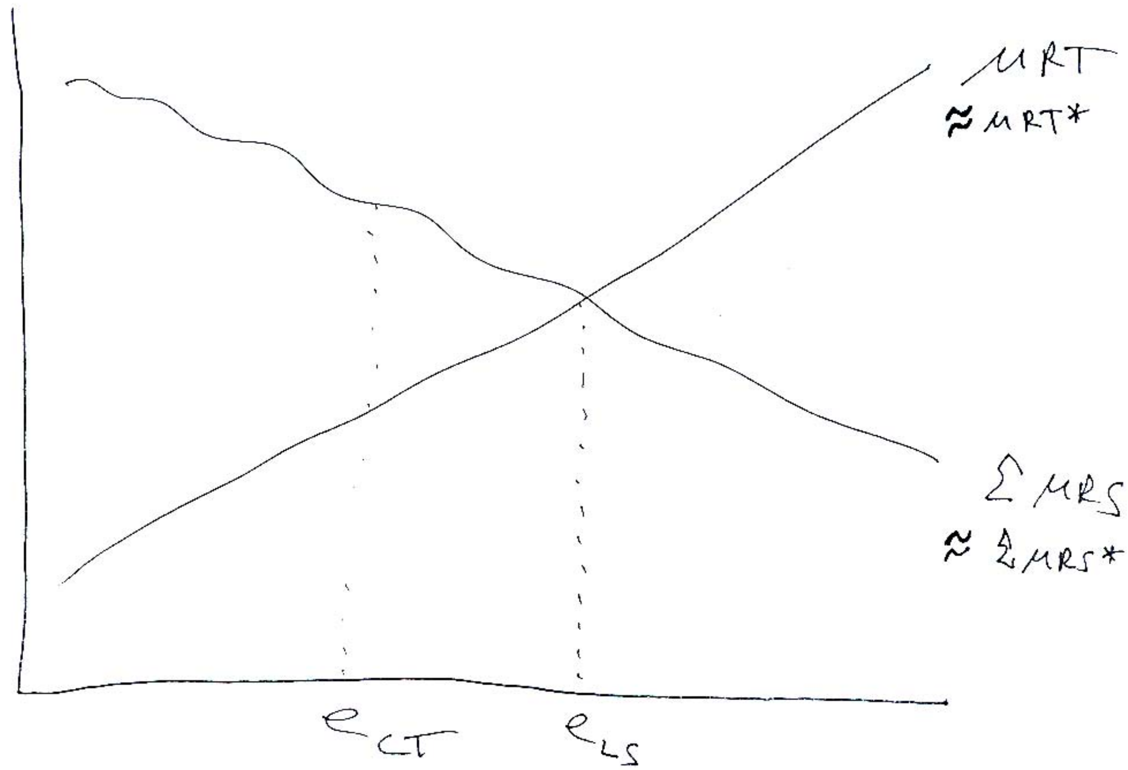
$$e_{CT} < e_{LS}$$

Well, those are a lot of assumptions!

(c) Ever since, people have looked for an example with higher provision with commodity taxation than at the first-best.

Gronberg and Liu claim to have such an example. Their analysis is based on the analysis of marginal excess burden, however.

We will have to put off further discussion of this issue until we have developed that machinery.


MRT
≈ MRT*
Σ MRS
≈ Σ MRS*
e_CT
e_LS