Lecture 27

Epple-Filimon-Romer (1984); see also 1983 and 1993

1. Overview

   As a point of reference, consider this an extension of the Epple-Zelenitz model.

   (a) Income is still exogenous.
   However, individuals now differ by income.
   This will create heterogeneity within communities.

   (b) Government action within each jurisdiction is determined by voting.
   Voting is the method by which preferences are aggregated.

   (c) Voters are somewhat myopic when they vote.
   They use a particular rule of thumb to predict the effect of the public good on the gross price of housing.

   The intellectual precursor to this paper is the one by Rose-Ackerman, in the reader.

2. There are T communities and a continuum of consumers.

3. Direct preferences, budget constraint, demand

   (a) Preferences are over housing (h), publicly provided good (x), and composite commodity (b).
   Housing will be produced from land and composite commodity.

   (b) Convenient notation for preferences:

   $$U(x, h, b)$$

   All individuals are assumed to have the same direct utility function. Thus, demands are going to vary across individuals only through variations in income.
   U is assumed separable in x and (h, b):

   $$U(x, h, b)$$

   Thus, changes in public good supply does not affect the demand for housing by existing residents.
   Any change in demand is due to migration.

   (c) A typical individual is endowed with income y (they suppress superscripts).
   It is completely exogenous.

(d) Income is distributed uniformly on the interval $[1, 2]$. So, income quintiles are all the same size.

(e) Budget constraint:

$$y = ph + b$$

where p is the gross price of housing.

(f) Choosing h and b to maximize utility subject to the budget constraint gives the following results.

i. Lagrangean:

$$\mathcal{L} = U(x, h, b) + \lambda(y - ph - b)$$

Given separability, demands are:

$$h(p, y), \quad b(p, y)$$

Indirect utility is:

$$V(x, p, y) \equiv U[x, h(p, y), b(p, y)]$$

ii. The following properties will be used in a moment.
From the first order conditions:

$$\frac{\partial \mathcal{L}}{\partial b} = U_b - \lambda = 0$$

Therefore:

$$\lambda = U_b$$

From the envelope theorem,

$$V_x(x, p, y) = U_x[x, h(p, y), b(p, y)]$$
$$V_p(x, p, y) = -\lambda h = -h(p, y)U_b[x, h(p, y), b(p, y)]$$

4. Indifference curves "through a point": The single crossing condition

(a) Pick a point $(\bar{x}, \bar{p})$.

We want the indifference curve of the person with income y through this point.

Formally, this is:

$$\{(x, p)|V(x, p, y) = V(\bar{x}, \bar{p}, y)\}$$

Use the implicit function theorem to obtain the function $p(\cdot)$ for which this set is its graph. In general we will write:

$$p(x, y) \equiv p[x, y, V(\bar{x}, \bar{p}, y)]$$

suppressing the dependence on $(\bar{x}, \bar{p})$. Note that by construction:

$$p(\bar{x}, y) = \bar{p}, \quad \text{all } y$$

(b) Slope

From the implicit function theorem and the previous results,

$$\frac{\partial p}{\partial x} = -\frac{V_x}{V_p}\bigg|_{(x,p(x),y)} = \frac{U_x(\cdot)}{h(\cdot)U_b(\cdot)} \tag{1}$$

This is equation (2) in Epple-Filimon-Romer (1984). Obviously:

$$\frac{\partial p}{\partial x} > 0$$

(c) Curvature

It is useful to have strict concavity or strict convexity of $p(x, y)$.

Formally:

$$\begin{aligned} \frac{\partial^2 p}{\partial x^2} &= \frac{\mathrm{d}}{\mathrm{d}x}\left\{-\frac{V_x[x, p(x), y]}{V_p[x, p(x), y}\right\} \\ &= -\frac{1}{V_p^3}(V_x^2 V_{pp} - 2V_p V_x V_{px} + V_p^2 V_{xx}) \end{aligned}$$

Since $V_p < 0$ we need:

$$V_x^2 V_{pp} - 2V_p V_x V_{px} + V_p^2 V_{xx} < 0$$

In footnote 7 of the paper (p. 292), assumptions (i), (ii), and (iii) assure that this holds. In footnote 1 of their 1983 paper they actually give the proof.

Figure, top panel

(d) Single crossing

Fix two levels of income:

$$y_1 > y_0$$

By construction:

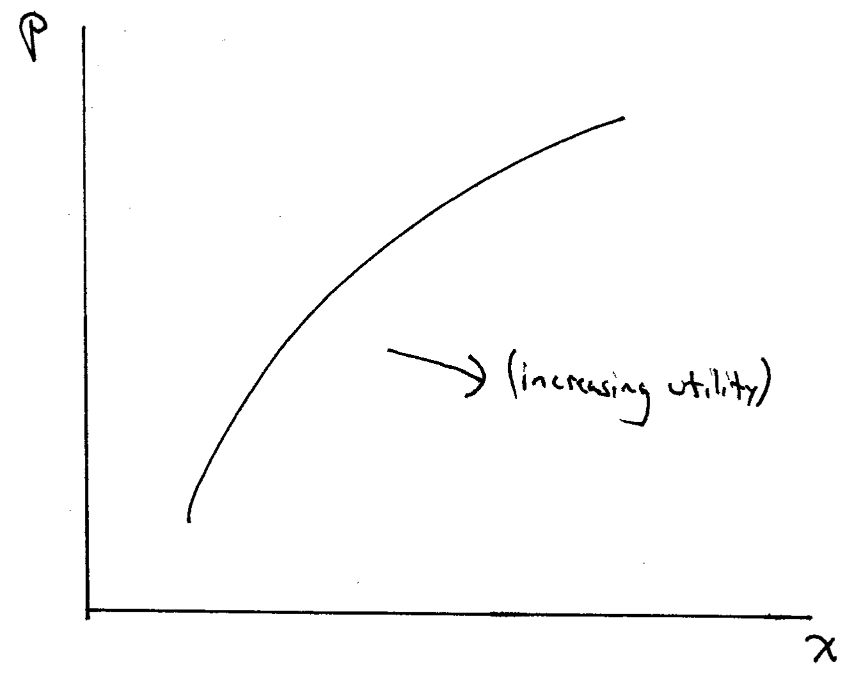
$$p(\bar{x}, y_0) = p_1(\bar{x}, y_1) = \bar{p}$$

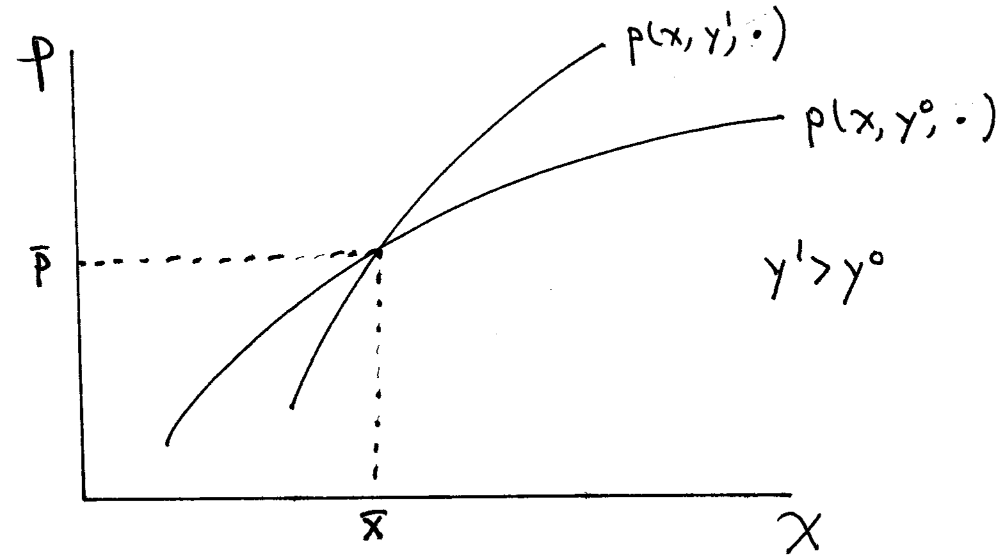
So, the indifference curves cross at $(\bar{x}, \bar{p})$.

Note that the curve at income y_1 must correspond to higher utility than the one at income y_0.

Epple-Filimon-Romer want to assure that the indifference curve defined at y_1 has a steeper slope at $(\bar{x}, \bar{p})$ than does the indifference curve defined at y_0. Formally:

$$\frac{\partial^2 p(\bar{x}, y_1)}{\partial x^2} > \frac{\partial^2 p(\bar{x}, y_0)}{\partial x^2}$$


p
(increasing utility)
x


p
p(x, y¹; ·)
p(x, y°; ·)
p̄
y¹ > y°
x̄
x

This is assured by restricting the cross partial:

$$\frac{\partial^2 p}{\partial x \partial y} > 0, \quad \text{all } x, y \tag{2}$$

i. Recall (1) above. The restriction in (2) amounts to the requirement that the income elasticity of MRS_{xb} exceeds the income elasticity of housing demand.

ii. There is a (difficult) discussion of this and other issues in Bucovetsky, "Optimal Jurisdictional Fragmentation and Mobility," *Journal of Public Economics*, 1981.

Figure, bottom panel

(e) Epple-Filimon-Romer express single-crossing by defining the function M:

$$M(x, p, y) \equiv \left.\frac{\mathrm{d}p}{\mathrm{d}x}\right|_{\bar{V}}$$

and then assuming:

$$\frac{\partial M(x, p, y)}{\partial y} > 0 \tag{3}$$

5. Housing Market

(a) Housing is produced from land and capital.

The discussion of housing supply in Epple-Zelenitz and Hoyt applies. In particular, the housing producer treats land as an input in perfectly inelastic supply, capital as an input in perfectly elastic supply, sees land in inelastic supply, the views the (net) price of housing as fixed. The producer is a profit maximizer but is not allowed to act like a monopolist.

(b) The housing supply function is the same as in Hoyt. In their notation, aggregate housings supply is:

$$H_s(p_h)$$

(c) Community boundaries are fixed and no new communities may enter.

6. Fiscal choices of local governments

(a) The local government chooses x, the per-capita amount of public service.

(b) Non-residents are excluded from consuming x.

(c) The total cost of providing x to each of the N members of the community is:

$$c(x, N)$$

The marginal cost is:

$$c_x(x, N)$$

$c(x, N)$ is assumed weakly convex in x.

More assumptions are added later.

(d) The cost is met through a proportional property tax, t, on the net price of housing.

Thus, gross and net price are linked by:

$$p = p_h(1 + t)$$

Budget balance requires:

$$tp_hH = c(x, N)$$

7. The local political process

   (a) x is determined by majority rule.

8. Equilibrium.

   (a) An intercommunity equilibrium is an allocation of individuals to communities and goods to individuals such that:

      i. Each individual lives in one and only one community.
      ii. Each community has positive population.
      iii. No individual can increase utility by moving.
      iv. Each community is in internal equilibrium.

   (b) An internal equilibrium is an allocation such that:

      i. The housing market clears.
      ii. The community budget is balanced.
      iii. There is political equilibrium.
      iv. No individual can, by altering his consumption bundle, increase his utility.

9. Internal Equilibrium

   (a)
$$tp_hH = c(x, N)$$

$$p = p_h(1 + t)$$

These give:

$$P(x) = p_h + \frac{c(x, N)}{H} \tag{4}$$

(b) THEY ASSUME voters take p_h, N, H as fixed when they vote. This is a kind of myopia.

(c) Optimal x for an individual.

For given income (that is to say, given a particular individual), equation (4) and $V(x,p,y)$ define the optimal x for that person.

Formally, use $V(x,p,y) = \bar{V}$ to define $p(x,y)$ (ignore the abuse of notation).

Strict concavity of $p(x,y)$ in x and weak convexity of $P(x)$ assure a unique tangency. This gives the optimal x (and the associated p) for each individual.

From the derivatives we know that the tangency is given by:

$$M(x,p,y) = \frac{c_x(x,N)}{H} \tag{5}$$

(d) Preference aggregation

Concavity of $p(x,y)$ and weak convexity of $P(x)$ imply preferences for all individuals over x are single peaked.

Thus, the median of the most preferred x's defeats any alternative in a pairwise vote.

Furthermore, the restriction in equation (2)(or (3)) assures that a voter's most preferred level of x on $P(x)$, the perceived community budget constraint, increases with y.

Thus, the median of the most preferred x's is held by the person with median income.

Let $\tilde{y}$ denote median income. Then the x that is the majority rule equilibrium satisfies:

$$M(x,p,\tilde{y}) = \frac{c_x(x,N)}{H} \tag{6}$$

(e) For internal equilibrium, p and p_h must also clear the housing market:

$$H_d(p) = H_s(p_h)$$

This holds simultaneously with (4), so the government's budget balances.

(f) The assumptions so far do not assure that a solution to all of these equations exists. EFR impose additional assumptions to assure that one does. These are the remaining assumptions, numbered (iv)-(vi), in footnote 7:

   i. The slope of an indirect indifference curve becomes infinite as x goes to zero.

   ii. The slope of an indirect indifference curve goes to zero as p becomes infinite.

iii. The price elasticity of housing demand cannot exceed one in absolute value: that is to say, housing demand cannot be too elastic.

Also, $c(0, N)$ cannot be "too large."

[HOWEVER, when they later discuss inter-community equilibrium, they require $c(0, N)$ not be "too small." This issue is technical and handled better in the 1993 paper.]

(g) Illustration of an internal equilibrium.

Figure

Notice that without separability of preferences, aggregate housing demand would depend on x. Aggregate housing demand would not necessarily remain fixed as x varied.

10. Inter-community equilibrium

(a) Three necessary conditions for inter-community equilibrium.

i. Stratification: each community is formed of individuals with incomes in a single interval.

ii. Boundary Indifference: the border consumer between adjacent communities is indifferent.

iii. Ascending Bundles: Let y^i denote the highest income person in i and y^j denote the highest income person in j. Then $y^j > y^i$ if and only if

$$x^j > x^i$$

and

$$p^j > p^i$$

(b) The following graph illustrates an inter-community equilibrium.

Figure, top panel

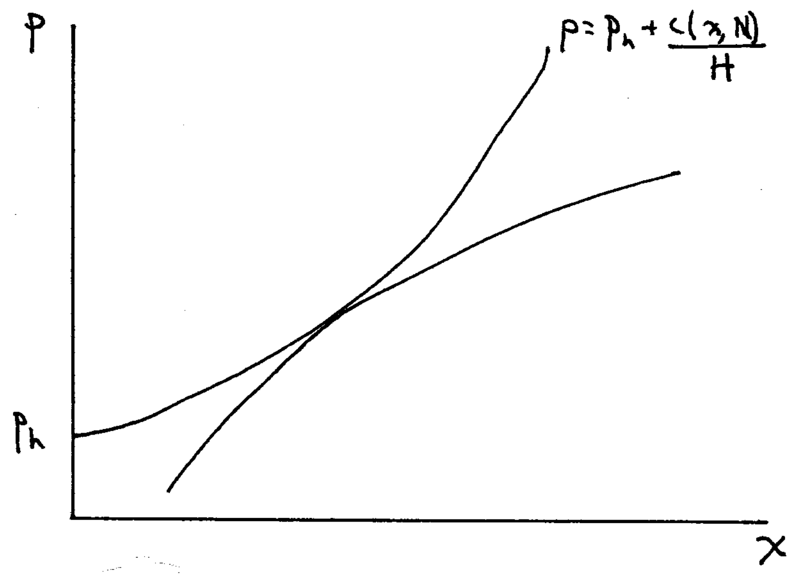
(c) The following graph shows how "ascending bundles" may fail to hold (and therefore inter-community equilibrium may fail to exist) despite all other restrictions made so far holding, including stratification and boundary indifference.

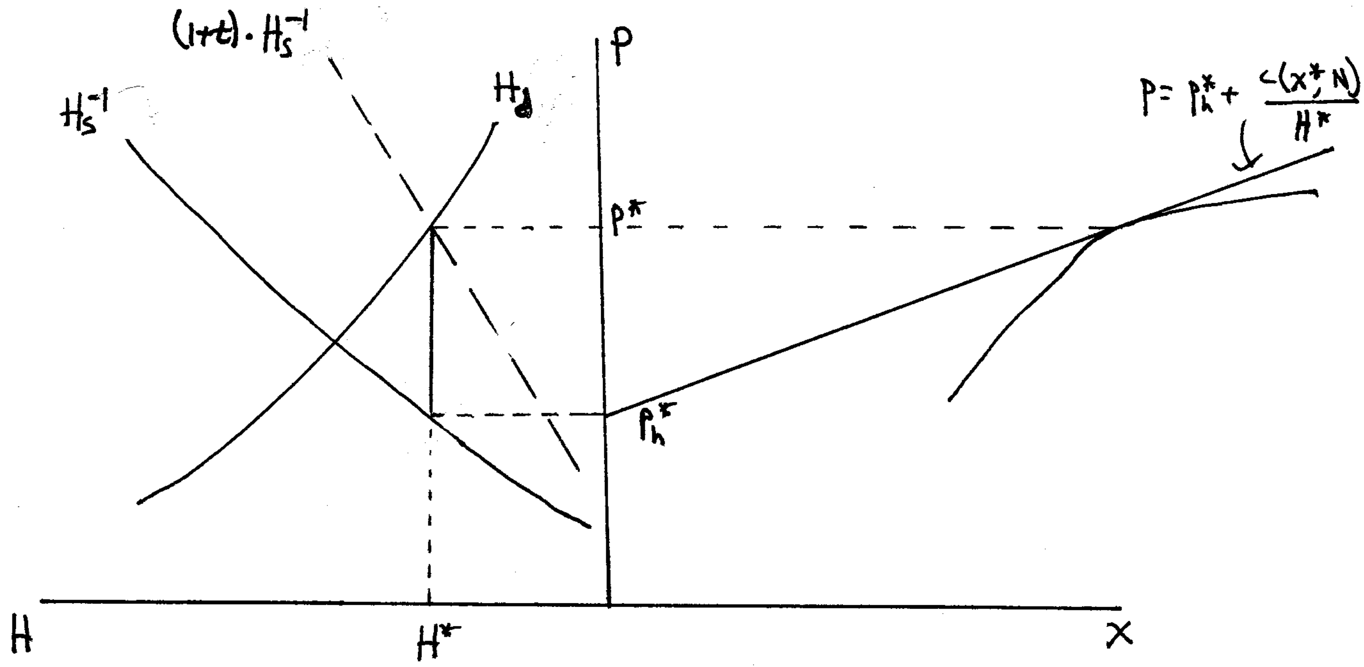
Figure, bottom panel

(d) How can we rule out the case just illustrated?

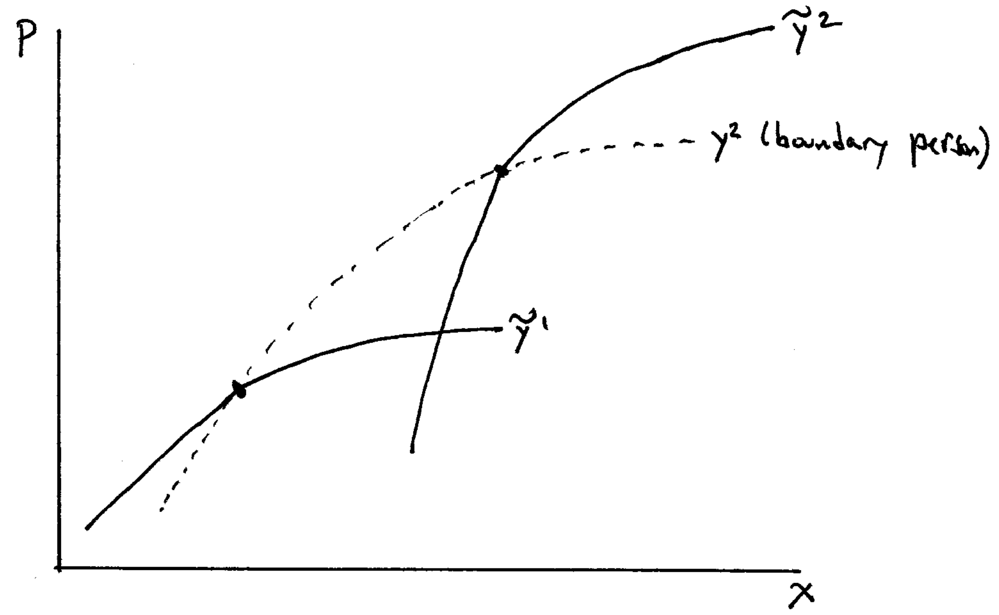
The first step is to impose a functional form assumption on the cost of the public good:

$$c(x, N) = c_0 + c_1 x N, \quad c_0 \geq 0, \ c_1 > 0 \tag{7}$$

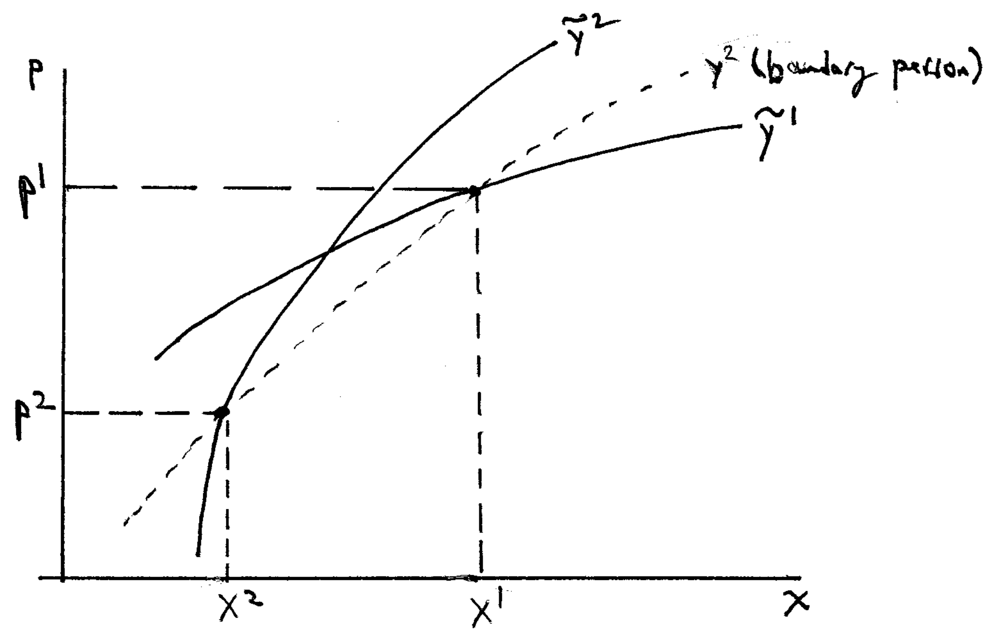

p
p = p_h + c(x, N) / H
p_h
x


(1+t)·H_s^-1
H_s^-1
H_d
p
p*
p_h*
p = p_h* + c(x*, N) / H*
H
H*
x

Equilibrium



Not Equilibrium



We have two communities:

$$[y^1, y^2], [y^2, y^3]$$

with median incomes:

$$\tilde{y}^1 < y^2 < \tilde{y}^2$$

and internal equilibria:

$$(x^1, p^1), (x^2, p^2)$$

Suppose stratification and boundary indifference hold. We now want to show that ascending bundles holds.

By boundary indifference, the individual with income y^2 is indifferent between the two internal equilibria. This means (x^1, p^1) and (x^2, p^2) lie on the same indifference curve for this person. One thing this implies is that higher p cannot be associated with lower x. Thus, the only possible cases are:

i. $(x^1, p^1) << (x^2, p^2)$

ii. $(x^1, p^1) >> (x^2, p^2)$

iii. $(x^1, p^1) = (x^2, p^2)$

In the first case ascending bundles holds, so we are done.

We now show that the other two cases are not possible.

In these two cases, we have $p^1 \geq p^2$. By strict normality of housing demand and the fact that all incomes in community 1 are lower than incomes in community 2:

$$\frac{H_d^1(p^1)}{N^1} < \frac{H_d^2(p^2)}{N^2}$$

Rearranging and the assumption c_1 positive gives:

$$\frac{c_1 N^1}{H_d^1(p^1)} > \frac{c_1 N^2}{H_d^2(p^2)} \tag{8}$$

Using equation (6) for internal equilibrium and the function form in (7) gives:

$$M(x^1, p^1, \tilde{y}^1) = \frac{c_1 N^1}{H_d^1(p^1)}$$

$$M(x^2, p^2, \tilde{y}^2) = \frac{c_1 N^2}{H_d^2(p^2)}$$

With (8) this gives:

$$M(x^1, p^1, \tilde{y}^1) > M(x^2, p^2, \tilde{y}^2) \tag{9}$$

We cannot have $(x^1, p^1) = (x^2, p^2)$ since then we would violate the requirement that indifference curves through a common point become steeper with income.

So, suppose $(x^1, p^1) >> (x^2, p^2)$. Since both lie on the same indifference curve for individual y^2, concavity of indifference curves gives:

$$M(x^1, p^1, y^2) < M(x^2, p^2, y^2)$$

This with increasing slope in incomes give:

$$M(x^1, p^1, \tilde{y}^1) < M(x^1, p^1, y^2) < M(x^2, p^2, y^2) < M(x^2, p^2, \tilde{y}^2)$$

This contradicts (9).

(e) The remaining question is a technical one, how to guarantee the existence of (an internal) boundary indifferent individual. That is to say, to guarantee that all communities are occupied.

This is assured by the final restrictions, on page 296.