Lecture 26

Wilson (1997)
Wilson (2003)

1. Wilson (1997) raises the question, to what extent does the property tax really "price" the congestion caused by migration?

   Recall that Hoyt (1991) argues that small competitive communities would use only the property tax and not some combination of property and land taxes because the property tax does a better job of pricing congestion.

   He also argues (without using the term) that there is a Prisoner's Dilemma here: all communities would be better off if they were denied permission to use the property tax and had to use only the land tax. The incentives at work in equilibrium do not lead to efficiency.

2. Wilson (1997, 2003) is concerned with two parts of this claim.

   Wilson (1997) asserts that neither the property tax nor the land tax prices congestion "properly" from a normative (as opposed to a positive) point of view.

   Second, Wilson (2003) implies that Hoyt's welfare analysis is wrong. Wilson argues that small competitive communities are better off with the property tax than the land tax. Wilson never uses these words, but this is the obvious implication.

3. There is an interesting subtlety in the meaning of the Samuelson condition in this model.[1] In a sense we have already dealt with this issue, we just didn't discuss it explicitly.

   (a) In an earlier lecture we considered the following problem for 2 jurisdictions and identical individuals. We had

   $$U_j(G_j, X_j^i), \quad j = 1, 2$$

   $$(G_1, X_1^i) = (G_2, X_2^i) \Longrightarrow U_1(G_1, X_1^i) = U_2(G_2, X_2^i)$$

   $$f_j(N_j) = X_j + C_j(G_j, N_j)$$

   $$N_1 + N_2 = \bar{N}$$

[1] Actually, there is a technical problem in writing down the pure planner problem in the Epple-Zelenitz model since the capital part isn't closed. What is the aggregate resource constraint on the capital stock? Wilson closes his model by endowing people with capital. This presents certain technical problems we mentioned in the last lecture. We will not resolve these issues today.

The Pareto problem involved choosing G_j, X_j^i, and N_j for $j = 1, 2$ to maximize $U(G_1, X_1^i)$ subject to the constraints:

$$U_2(G_2, X_2^i) = \bar{U}_2$$

$$\sum_j f_j(N_j) - \sum_j N_j X_j^i - \sum_j C_j(G_j, N_j) = 0$$

$$N_1 + N_2 = \bar{N}$$

One of the results was "the Samuelson condition" for both communities:

$$N_j \frac{U_{j1}}{U_{j2}} = \frac{\partial C_j}{\partial G_j}, \quad j = 1, 2$$

(b) Now, think again about the problem we solved. We *assumed* all agents in all regions would be given the same allocation.

This restriction is implied if G is non-excludable. In that case, the restriction is really just part of feasibility.

If G is excludable – and a good is excludable is if it is private or somewhat rival in consumption – then this restriction is really an additional requirement. We are limiting attention to efficient allocations that provide equal treatment *in consumption* within each region.

If G is a publicly provided private good, then efficiency still requires:

$$\frac{U_{j1}}{U_{j2}} = \text{MRT}_j^{GX}$$

Thus, in this case, we must have:

$$\frac{\partial C_j}{\partial G_j} = (N_j)\text{MRT}_j^{GX}$$

This makes sense. The MRT_j^{GX} tells us how much of good X must be foregone for an extra unit of G. In order to give an extra unit of G to every individual when G is a publicly provided private good, we must forego as many units of X as there are individuals.

The conclusion is that "the Samuelson condition" holds whether G is a public good or private good or something in between. The interpretation of $\frac{\partial C_j}{\partial G_j}$ depends, however, on the nature of G.

4. Wilson (1997) really operates with the same model as Hoyt (1991). Only the nation is different (radically different, unfortunately). We will stay with the notation in which we developed Hoyt. Hoyt and Wilson essentially use the model of Epple-Zelenitz, except the property tax is a unit tax instead of an ad valorem tax. In other words, the gross price of housing is $p + \tau$ instead of $p(1 + t)$.

5. Briefly, the key equations are:

$$F(K, L), \quad L = \bar{L}$$

Preferences and budget constraint:

$$U(g, h, b)$$

$$y = (p + \tau)h + b$$

Utility maximization gives *housing demand per-capita*:

$$h_d(g, p + \tau)$$

Profit maximization gives:

$$K_d(p)$$

Housing supply *per unit of land* is:

$$h_s(p) \equiv \frac{F[K_d(p), L]}{L}$$

Define:

$$p_L(p) \equiv \frac{1}{L}\left\{pF[K_d(p), L] - p_K K_d(p)\right\}$$

The equilibrium conditions give us *all* endogenous quantities as functions of (τ, g). The analysis focuses on $p(\tau, g)$ and $N(\tau, g)$ because the chain rule gives all of the others. For example:

$$\begin{aligned} \frac{\partial p_L}{\partial p} &= \frac{1}{L}\left\{F[K_d(p)] + pF_K(K_d)' - p_K(K_d)'\right\} \\ &= \frac{F}{L} \end{aligned}$$

In equilibrium (a slight abuse of notation here):

$$p_L(\tau, g) \equiv p_L[p(\tau, g)]$$

so:

$$\begin{aligned}\frac{\partial p_L}{\partial g} &= \frac{\partial p_L(p)}{\partial p}\frac{\partial p}{\partial g}\\ &= -\frac{F}{L}\frac{\partial p}{\partial g}\end{aligned}$$

It is also straightforward to establish:

$$\frac{\partial p}{\partial g} = \frac{U_g}{U_b}\frac{1}{h_d}, \quad \frac{\partial p}{\partial \tau} = -1, \quad \frac{\partial h_d[g, p(\tau, g) + \tau]}{\partial \tau} = 0$$

We show the first; we established the second last time and the third is then immediate.

(a) Utility maximization gives:

$$\max_{h,b} \; U(g, h, b) \qquad \text{subject to:} \qquad y = (p + \tau)h + b$$

The Lagrangian is:

$$\mathcal{L} = U(g, h, b) + \lambda[(p + \tau)h + b - y]$$

which gives, among other things:

$$U_b = -\lambda$$

Utility-taking gives:

$$V(p + \tau, g) = \bar{V}$$

so:

$$\begin{aligned}\frac{\partial p}{\partial g} &= -\frac{\partial V/\partial g}{\partial V/\partial p}\\ &= -\frac{U_g}{\lambda h_d}\\ &= \frac{U_g}{U_b}\frac{1}{h_d}\end{aligned}$$

6. Wilson's equation (9).

Recall the optimization problem in Hoyt. The objective is to maximize net land rent:

$$\max_{\tau, T, g} \; p_L(\tau, g)L - TL \qquad \text{subject to:} \qquad N(\tau, g)\{\tau h_d[g, p(\tau, g) + \tau] - g\} + TL = 0$$

As a technical matter, Wilson (1997, 2003) considers the exact same problem. All he does is (a) substitute the budget constraint into the objective function, (b) use a more general cost function, and (c) focus on the derivative with g instead of with τ.

Making the substitution gives:

$$p_L(\tau, g)L + N(\tau, g)\{\tau h_d[g, p(\tau, g) + \tau] - g\}$$

Multiplying through:

$$p_L(\tau, g)L + N(\tau, g)\tau h_d[g, p(\tau, g) + \tau] - N(\tau, g)g$$

Define:

$$C(g, N) \equiv Ng$$

Then:

$$p_L(\tau, g)L - C[g, N(\tau, g)] + N(\tau, g)\tau h_d[g, p(\tau, g) + \tau]$$

This is the objective function in Wilson.

Implicitly, Wilson's equations utilize the direct and indirect effects of a change in g on per-capita housing demand. Formally, define:

$$\frac{\mathrm{d}h_d}{\mathrm{d}g} \equiv \frac{\mathrm{d}}{\mathrm{d}g}\, h_d[g, p(\tau, g) + \tau]$$

Differentiating the objective function with g then gives:

$$L\frac{\partial p_L(p)}{\partial p}\frac{\partial p}{\partial g} - C_g - C_N\frac{\partial N}{\partial g} + \frac{\partial N}{\partial g}\tau h_d + N\tau\frac{\mathrm{d}h_d}{\mathrm{d}g} = 0$$

Rearranging gives:

$$C_g + [C_N - \tau h_d]\frac{\partial N}{\partial g} - \tau N\frac{\mathrm{d}h_d}{\mathrm{d}g} = L\frac{\partial p_L(p)}{\partial p}\frac{\partial p}{\partial g}$$

Using our previous results:

$$L\frac{\partial p_L(p)}{\partial p}\frac{\partial p}{\partial g} = F\frac{U_g}{U_b}\frac{1}{h_d} = \left(\frac{F}{Nh_d}\right)N\frac{U_g}{U_b} = N\frac{U_g}{U_b}$$

where the last step uses the fact aggregate housing supply equals aggregate housing demand.

This gives his equation (9):

$$C_g + [C_N - \tau h_d]\frac{\partial N}{\partial g} - \tau N\frac{\mathrm{d}h_d}{\mathrm{d}g} = N\frac{U_g}{U_b} \tag{1}$$

7. Wilson's Proposition 1.

(a) Suppose only a land tax is available.
Then $\tau = 0$ and (1) reduces to:

$$C_g + C_N \frac{\partial N}{\partial g} = N \frac{U_g}{U_b} \tag{2}$$

(b) If only a property tax is available, then $T = 0$ and the constraint in the problem becomes:

$$\tau N(\tau, g) h_d[g, p(\tau, g) + \tau] = C[g, N(\tau, g)]$$

Rather than repeat the optimization, Wilson makes the following argument. The argument also goes through if T is some exogenously fixed value.

Use the constraint to obtain the function $\tau(g)$. Substitute this into the objective function to obtain the unconstrained problem of maximizing. $p_L[p(\tau(g), g)]L$.

A stationary point of the total derivative of p_L with g is a stationary point of the *total* derivative of p with g. Thus, taking into account all effects, the equilibrium net price of housing does not change. More precisely, the change in g shifts the aggregate housing demand curve, but this curve moves along a fixed aggregate housing supply curve. The fact that the equilibrium net price of housing does not change therefore means that the equilibrium quantity of housing does not change. Furthermore, since the housing market must clear, this means:

$$\frac{\mathrm{d}}{\mathrm{d}g} \left\{ N[\tau(g), g] h_d[g, p(\tau(g), g) + \tau(g)] \right\} = 0$$

Therefore:

$$-h_d \frac{\mathrm{d}N}{\mathrm{d}g} = N \frac{\mathrm{d}h_d}{\mathrm{d}g}$$

If we reinterpret (1) to recognize the absence of the land tax, so τ must adjust to balance the budget, we can use the previous result and again obtain (2).

(c) We have seen that a characteristic of the choice of g that maximizes land-rent when revenue is raised (at the margin, at least) by the property tax is that a small change in g from this value would produce no change in the equilibrium quantity of housing.

The change in g in general causes more people to enter the region (see the discussion of his equation (14)), but their demand for housing is offset by lower per-capita housing demand from existing residents.

This has a further implication. An increase in g must increase the equilibrium size of the government budget. This is $C\{g, N[\tau(g), g]\}$, so if people enter then total costs must increase, whatever the properties of g as a public or private good. Although the aggregate quantity of housing does not change, government revenue increases because the tax rate adjusts.

Wilson seems to miss this. He states, "[A]ny revenue gain from the property tax payments collected from new residents is exactly offset by the lost revenue from lower per-capita housing demand" (p. 214). This does not seem quite right. The cost of g increases and the government's budget is balanced. Since the tax base is constant, the tax rate must increase.

8. Wilson (2003) builds on all of this.

   He agrees with Hoyt that unit taxes on property capture congestion better than unit taxes on land do. He argues, however, that an exogenous shift away from a land tax toward the property tax raises welfare. A property tax is preferable to a land tax. This is the opposite of what Hoyt argued.

   Wilson's explanation is essentially that the shift *reduces* fiscal competition. Moving to the property tax would reduce the tendency of an increase in g to increase the local population. Each region would recognize this and increase g. This is the right incentive: equation (1) means that the marginal benefit of the increase exceeds the marginal welfare cost, so at least locally it must be the case that welfare would rise from increasing g.[2]

[2] The global argument faces the "rules" versus "levels" problem we saw when discussing Atkinson-Stern.