Lecture 24

Stiglitz (1983)
Epple-Zelenitz (1981) and Henderson (1985)

Stiglitz (1983)

1. Direct utility is defined over private good (say X), public good G, and land T. Each individual solves:

   $$\max_{X,T} \; U(X,G,T)$$
   $$\text{subject to:} \qquad X + rT = Y$$

   where r is the gross price of land and Y is exogenous income. The Lagrangian is:

   $$\mathcal{L} = U(X,G,T) + \alpha(X + rT - Y)$$

   This gives demand for private good and land and indirect utility:

   $$X(Y,G,r), \;\; T(Y,G,r)$$

   $$V(Y,G,r)$$

   From the first order condition:

   $$\frac{\partial U}{\partial X} = -\alpha$$

   so from the envelope theorem:

   $$\frac{\partial V}{\partial r} = \alpha T = -\frac{\partial U}{\partial X}T$$

2. Suppose the government chooses public good to maximize land rent (net of the cost of public goods), but recognizes that the population and land rent are endogenous. Both adjust so maintain a fixed level of utility for residents and clear the land market. More precisely,

   $$V(Y,G,r) = \bar{V}, \;\; nT(Y,G,r) = \bar{T}$$

simultaneously determine the functions:

$$r(G), n(G)$$

Then the government's problem is:

$$\underset{G}{\text{Max}} \;\; r(G)\bar{T} - G$$

The first order condition is:

$$\frac{\partial r}{\partial G}\bar{T} = 1, \;\; so \frac{\partial r}{\partial G} = \frac{1}{\bar{T}}$$

Differentiating:

$$V[Y, G, r(G)] \equiv \bar{V}$$

gives:

$$\frac{\partial V}{\partial G} + \frac{\partial V}{\partial r}\frac{\partial r}{\partial G} = 0$$

Differentiating:

$$n(G)T[Y, G, r(G)] \equiv \bar{T}$$

gives:

$$\frac{\partial n}{\partial G}T + n\left(\frac{\partial T}{\partial G} + \frac{\partial T}{\partial r}\frac{\partial r}{\partial G}\right) = 0$$

Re-writing these two as a system of equations gives:

$$\begin{bmatrix} \frac{\partial V}{\partial r} & 0 \\ n\frac{\partial T}{\partial r} & T \end{bmatrix} \begin{bmatrix} \frac{\partial r}{\partial G} \\ \frac{\partial n}{\partial G} \end{bmatrix} = \begin{bmatrix} -\frac{\partial V}{\partial G} \\ -n\frac{\partial T}{\partial G} \end{bmatrix}$$

Therefore:

$$\frac{\partial r}{\partial G} = \frac{\begin{vmatrix} -\frac{\partial V}{\partial G} & 0 \\ -n\frac{\partial T}{\partial G} & T \end{vmatrix}}{\begin{vmatrix} \frac{\partial V}{\partial r} & 0 \\ n\frac{\partial T}{\partial r} & T \end{vmatrix}} = -\frac{\partial V/\partial G}{\partial V/\partial r} = \frac{\partial V/\partial G}{T(\partial U/\partial X)}$$

Using the first order condition and this result gives the Samuelson condition:

$$\frac{1}{nT} = \frac{1}{\overline{T}} = \frac{\partial r}{\partial G} = \frac{\partial V/\partial G}{T(\partial U/\partial X)}$$

Therefore:

$$n\frac{\partial V/\partial G}{\partial U/\partial X} = 1$$

Epple-Zelenitz (1981) and Henderson (1985)

1. Epple and Zelenitz ask, "Does Tiebout need politics?" Their answer is, "yes." Here's the idea.
   (a) Suppose:
      i. Local governments want to maximize local government profits, or "fiscal surplus." This is the excess of tax revenue over expenses on publicly provided goods.
      ii. Residents are costlessly mobile.

   (b) The question is, what happens to the fiscal surplus as the number of jurisdictions gets large?

   Epple and Zelenitz show that it does *not* go to zero:

   > "Competition among governments is not equivalent to competition among firms [which are also profit maximizers facing "mobile" consumers]. A further implication is that the political choice process can matter."

   (c) The intuition behind the result is that there are fixed factors, like land, that are immobile. "Bad politics" can exploit them in equilibrium.

   This is *not* obvious. The return to immobile factors depends on the mobile factors. Land is used to construct housing, but neither has any value if no one wants to locate in the community.

   So it is certainly not obvious that the immobile can really be "exploited" in equilibrium. Epple and Zelenitz show that they can be.

2. Henderson argues that Epple and Zelenitz's model is flawed in two ways.
   (a) First, he argues that maximizing the "fiscal surplus" is not the most intuitive objective function for a local government.
      i. Landowners are likely to have some control over what the government does. They are not sitting around waiting to be exploited. If they are in control, then they probably want to maximize this fiscal surplus together with land rent.
      ii. Maximizing this objective function implies that the fiscal surplus disappears as the number of communities becomes large. "Fiscal exploitation" is eliminated.

   (b) Second, the Epple-Zelenitz model, taken on its own terms, leaves $100 bills on the table. Entrepreneurial activity by landowners would tend to eliminate fiscal exploitation.

i. In the long run, landowners can shift land to other uses besides housing, and may even shift land to other communities (by changing the community boundary). Also, new communities could form.

> "Bad politics are not possible because the landowners will collectively refuse to allocate their land in any community to those attempting to usurp their incomes."

ii. These activities will serve to eliminate "fiscal exploitation."

(c) He concludes that, in the long-run, Tiebout ("efficiency") does not need politics: population movement and land use adjustment lead to the same outcome whether there is no politics, good politics, or bad politics. In this outcome there is no fiscal surplus.

(d) Note: Henderson seems to be especially fond of his second critique. However, there is no formal modeling of the process he describes. This raises the question of whether the story he tells really captures all of the incentives.

Of course, if transactions costs are zero, then (with the right specification of property rights) we would expect fiscal exploitation to disappear. That is just the Coase theorem.

3. These papers are at the foundation of a large literature. They have a common formal structure but vary in their assumptions about:

(a) The objectives of local governments.

(b) The instruments available to local governments.

(c) The perspective of the local government, as defined by the equilibrium concept ("large numbers" or "small numbers").

In this overview we will focus on the "large numbers" case. This allows us to introduce the key results of both papers and their common formal structure while keeping the derivations short. We can then consider the "small numbers" analysis in Epple-Zelenitz (and later Hoyt) without getting lost (Henderson considers only the "large numbers" case).

Note that the "small numbers" case is more realistic in some situations, but not all. Regardless, people always want to know whether a substantively interesting qualitative result that holds with one concept will also hold in the other.

4. The Economy

(a) The total amount of land in the "metropolitan area" is $\bar{L}$.

There are J regions, $j = 1, ..., J$.

Each region has the same amount of land:

$$L = \bar{L}/J$$

(b) Each region has a constant returns to scale technology for producing housing out of land and capital:

$$F(K^j, L)$$

We assume that the technology is the same in all regions. Otherwise we cannot justify looking for symmetric equilibria.

(c) The specification of the agents in this model is a bit complicated. Some important decisions are made by agents who are absent. This is one reason that these models are not closed and that people disagree about the government objective function.

However, the only agents with utility functions are the $\bar{N}$ mobile individuals. They derive utility from publicly provided private good g, housing h, and composite commodity b:

$$U(g^j, h^j, b^j)$$

(d) Finally, before one can consider the planner problem, one must specify the exogenous aggregate resource constraints.

i. The amount of land per jurisdiction and the total number of people are two mentioned explicitly by Epple-Zelenitz and Henderson.

ii. There must also be an aggregate capital constraint and an aggregate constraint for the resource that creates composite commodity b and public good g. Epple-Zelenitz and Henderson do not discuss these things.[1]

[1] Hoyt does a little better with this. He assumes that there is a single commodity that can be directly consumed (as b), used to produce the public good, or used as capital in the production of housing (p. 354).

5. Completing the Model.

   (a) To complete the model we need to specify individual endowments, institutions and behavior. We can then turn to the equilibrium concept.

   (b) Each of the $\bar{N}$ agents are endowed with the same exogenous income I.[2] They have a budget constraint:

   $$I = p^j(1 + t^j)h^j + b^j$$

   where p^j is the net price of housing and t^j is an ad valorem tax on the net price of housing, i.e. – "the property tax."

   (c) The following agents are absentee: capital owners, landowners, and recipients of any fiscal surplus. For these agents, the regions are just places in which they can earn income. Epple-Zelenitz discuss this issue in footnote 9 and Henderson on page 253.

   (d) Epple-Zelenitz seem to think of the landowners in j as actually constructing and selling the housing. This allows them to suppress the price of land in the model.

   Henderson takes the same approach, but for a number of reasons he wants the price of land to appear explicitly. Under constant returns to scale, this price is just the return to landowners who construct and sell houses (house value less the cost of capital) divided by the quantity of land, since this return is also the quantity of land times the price of land.

   (e) The indeterminacy in "who the agents are" creates some of the indeterminacy in government behavior. Government behavior can be *derived* given a set of agents, some mapping from the actions of government to the utility of these agents, and a collective choice rule.

   Whether you think this approach in these papers is good or bad depends on whether you think:

      i. specifying a collective choice rule is also arbitrary, it just pushes the question back one level,
      ii. even in a democracy, the people actually engaged in the political process are a somewhat arbitrary subset of the set of agents, so various objectives are possible
      iii. good insights often follow from simple models without pure and complete microfoundations (sorry Marcus)

   In Epple-Zelenitz and Henderson it is assumed that the local governments control local public good and the property tax rate.

[2] Hoyt does not seem to want to treat I as exogenous (p. 355) but it isn't clear what alternative he has in mind.

The objectives considered are:

i. Maximize the fiscal surplus (reasonable when "bureaucrats" control taxing and spending).
ii. Maximize land rent plus fiscal surplus (reasonable when "developers" own the land and control taxing and spending).

6. Equilibrium.

Recall that L, p_K and I are exogenous.

(a) In all j, the values of h^j and b^j maximize the utility of a representative individual in each region, taking everything else as given:

$$\max_{h^j, b^j} \; U(g^j, h^j, b^j)$$
$$\text{subject to:} \qquad I = p^j(1+t^j)h^j + b^j$$

This defines the functions (we suppress I):

$$h_d^j[g^j, p^j(1+t^j)] \tag{1}$$

$$b^j[g^j, p^j(1+t^j)] \tag{2}$$

(b) In all j, K_d^j maximizes profits in the housing industry (modeled as a single price-taking firm):

$$\max_{K^j} \; p^j F(K^j, L) - p_K K^j$$

This defines the function:

$$K_d^j(p^j) \tag{3}$$

(c) The relationships in (1)-(3) (with constant returns to scale) define two additional variables, the equilibrium price of land (since the equilibrium quantity of land must be L) and housing supplied per unit of land:

$$p_L^j(p^j) \equiv \frac{1}{L}\left\{p^j F[K_d^j(p^j), L] - p_K K_d^j(p^j)\right\} \tag{4}$$

$$h_s^j(p^j) \equiv \frac{F[K_d^j(p^j), L]}{L} \tag{5}$$

(d) Market Clearing

In any equilibrium, the following market clearing conditions will have to hold.

i. Within each community, housing demand equals housing supply:

$$N^j h_d^j[g^j, p^j(1+t^j)] = L h_s^j(p^j), \quad j = 1, ..., J \tag{6}$$

ii. No individual wants to migrate. Using indirect utility, this is:

$$V[g^j, p^j(1+t^j)] = V[g^1, p^1(1+t^1)], \quad j = 2, ..., J \tag{7}$$

iii. Every individual resides somewhere:

$$\sum_j N^j = \bar{N} \tag{8}$$

(e) Large Number Case: Government Perspectives and Objectives

Consider the full list of endogenous variables, nine for each region j:

$$(p^j, p_L^j, K_d^j, h_d^j, h_s^j, b^j, N^j, t^j, g^j), \qquad j = 1, ..., J$$

The government in region j chooses t^j and g^j consistent with its objectives and taking into account the market clearing conditions it perceives. We first consider the large numbers perspective because it is simpler.

In the large numbers perspective, each government perceives the functions in (1)-(5) and the market clearing equations in (6)-(7) for its own region. For (7), this means it treats utility elsewhere as uniform and exogenous:

$$V[g^j, p^j(1+t^j)] = V^* \tag{9}$$

For each region j we now have nine variables and the seven equations (1)-(6) plus (9). The government in j chooses the remaining two, t^j and g^j, anticipating that the remaining seven will be determined by (1)-(6) plus (9). In principle the seven variables other than t^j and g^j are all functions of t^j and g^j and determined simultaneously by the seven equations.

We do not need the derivatives of all seven equations with respect to the government's choice variables. A sensible analysis focuses on a few expressions, like the elasticity of housing supply, that embody other derivatives (like the elasticity of capital demand) and which therefore never appear explicitly in the analysis.

Even if we did need them all, we would not want to use the general implicit function theorem. That ignores the simplicity of the system at hand. The choice of t^j and g^j entirely determines p^j through (9). This then determines capital demand, that determines housing supply, etc. Nesting the functions and using the chain rule will work better than building up big matrices. Once the derivatives of p^j and N^j are found the remainder follow easily.

For future reference, we write:

$$p^j(t^j, g^j), \quad N^j(t^j, g^j) \tag{10}$$

$$p^j_L(t^j, g^j) \equiv p^j_L[p^j(t^j, g^j)], \quad h^j_s(t^j, g^j) \equiv h^j_s[p^j(t^j, g^j)] \tag{11}$$

(f) Small Numbers Case: Government Perspectives and Objectives

In the small numbers perspective, we look for a (symmetric) Nash equilibrium in the choice variables of the governments. Each government moves simultaneously taking into account how the entire economic system adjusts in response.

More formally, each government perceives the functions in (1)-(5) for all regions j and the full set of market clearing equations in (6)-(9). It treats the choice variables of the governments in other regions as parameters.

So, it chooses t^j and g^j anticipating that $7J$ of the endogenous variables will be determined by the $5J$ equations in (1)-(5) plus the $2J$ equations given by $(6) - (8)$.

This now gives the functions:

$$p^j(t^1, ..., t^J, g^1, ..., g^J), \quad N^j(t^1, ..., t^J, g^1, ..., g^J) \tag{12}$$

$$p^j_L(t^1, ..., t^J, g^1, ..., g^J) \equiv p^j_L[p^j(t^1, ...t^J, g^1, ..., g^J)] \tag{13}$$

$$h^j_s(t^1, ..., t^J, g^1, ..., g^J) \equiv h^j_s[p^j(t^1, ...t^J, g^1, ..., g^J)]$$

(g) Government Objectives

The two objective functions are:

i. Maximize the fiscal surplus:

$$\pi^j_{FS}(t^j, g^j) = t^j p^j(.)Lh^j_s(.) - N^j(.)g^j$$

ii. Maximize net revenue:

$$\pi^j_{NR}(t^j, g^j) = p^j_L(.)L + t^j p^j(.)Lh^j_s(.) - N^j(.)g^j$$

Notice that just four functions that appear in the objective functions:

$$p^j(.), h^j_s(.), N^j(.), p^j_L(.) \tag{14}$$

The particular form of these functions depends on the perspective.

7. Derivatives of $p^j(t^j, g^j)$ (so, large numbers perspective).

(a) Using (9) and the implicit function theorem:

$$\frac{\partial p^j}{\partial t^j} = -\frac{\partial V^j/\partial t^j}{\partial V^j/\partial p^j} = -\frac{V_2^j p^j}{V_2^j(1+t^j)} = -\frac{p^j}{1+t^j} \tag{15}$$

This is equation (9a) in Epple-Zelenitz after sending J to infinity.

This means that the net price falls by the full amount of the tax and the gross price is unchanged. To see the latter explicitly:

$$\frac{\partial}{\partial t^j}[p^j(t^j, g^j)(1+t^j)] = \left(-\frac{p^j}{1+t^j}\right)(1+t^j) + p^j = 0$$

Figure 1

(b) For the other derivative:

$$\frac{\partial p^j}{\partial g^j} = -\frac{\partial V^j/\partial g^j}{\partial V^j/\partial p^j} = -\frac{V_1^j}{V_2^j(1+t^j)}$$

To evaluate this, recall the utility maximization problem:

$$\mathcal{L} = U(g^j, h^j, b^j) + \lambda[p^j(1+t^j)h^j + b^j - I]$$

At the solution, we have by the envelope theorem and the first order conditions:

$$V_1^j = U_g^j$$

$$V_2^j = \lambda h_d^j = -U_b^j h_d^j$$

Therefore:

$$\frac{\partial p^j}{\partial g^j} = \frac{U_g^j}{U_b^j}\frac{1}{h_d^j(1+t^j)} \tag{16}$$

This is equation (10a) in Epple-Zelenitz after sending J to infinity.

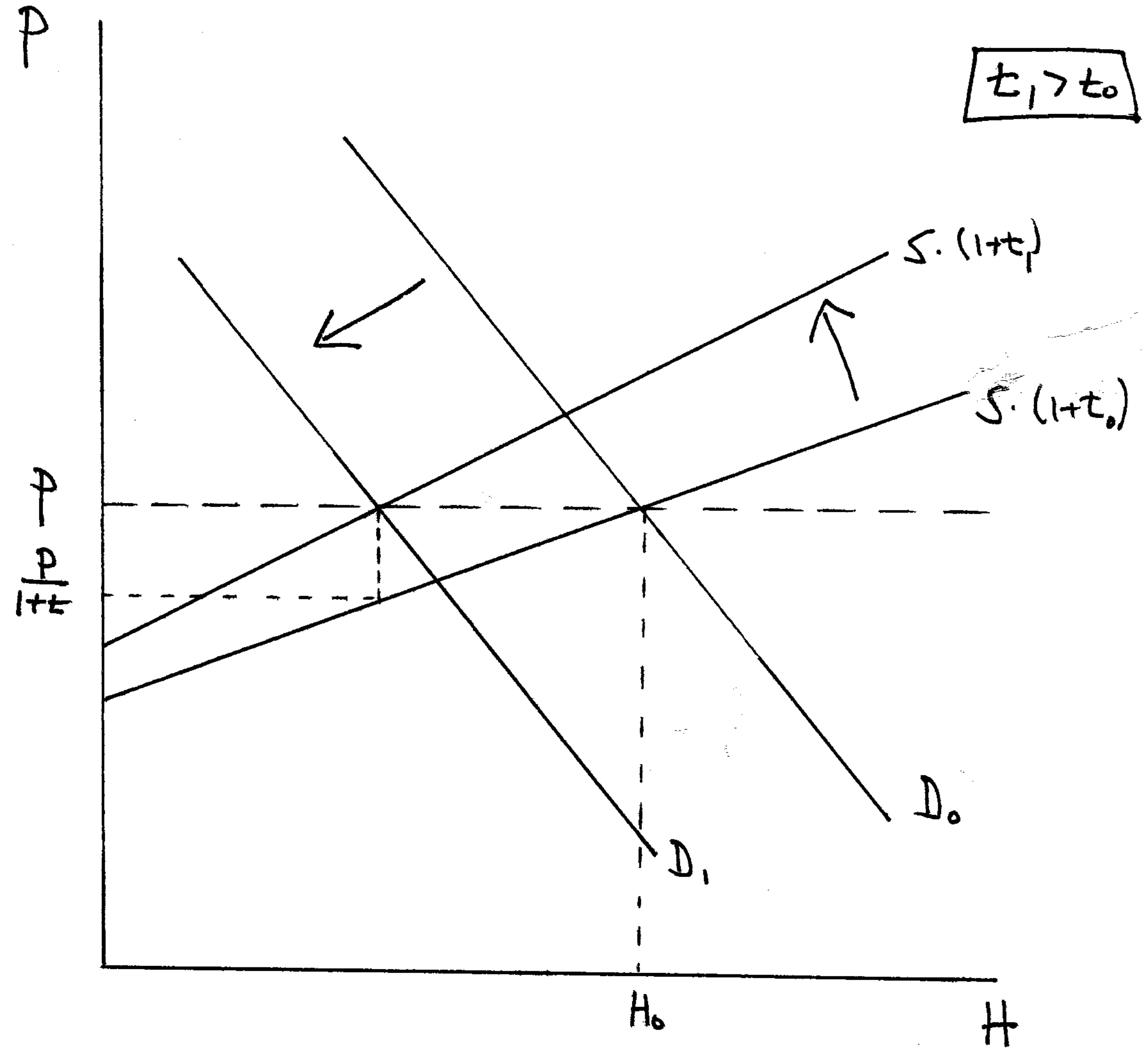

Figure 1

8. Derivatives of $N^j(t^j, g^j)$.

(a) In this case it is easiest to differentiate both sides of (6) after substituting in (10)-(11):

$$\frac{\partial N^j}{\partial t^j}h_d^j + N^j \frac{\partial h_d^j}{\partial p^j(1+t^j)}\frac{\partial p^j(1+t^j)}{\partial t^j} = L\frac{\partial h_s^j}{\partial p^j}\frac{\partial p^j}{\partial t^j}$$

The second term on the left is zero since the gross price of housing is unchanged. Using the derivative of p^j then gives:

$$\begin{aligned}\frac{\partial N^j}{\partial t^j}h_d^j = L\frac{\partial h_s^j}{\partial p^j}\left(-\frac{p^j}{1+t^j}\right) &= L\left(\frac{\partial h_s^j}{\partial p^j}\frac{p^j}{h_s^j}\right)\left(-\frac{h_s^j}{1+t^j}\right) \\ &= -L\theta^j\frac{h_s^j}{1+t^j} \\ &= -Lh_s^j\frac{\theta^j}{1+t^j} \\ &= -N^j h_d^j\frac{\theta^j}{1+t^j}\end{aligned}$$

So:

$$\frac{\partial N^j}{\partial t^j} = -\frac{N^j\theta^j}{1+t^j} \tag{17}$$

This is equation (11a) in Epple-Zelenitz after sending J to infinity.

(b) Again, differentiating both sides of (6) after substituting in (10)-(11):

$$\frac{\partial N^j}{\partial g^j}h_d^j + N^j\left[\frac{\partial h_d^j}{\partial g^j} + \frac{\partial h_d^j}{\partial p^j(1+t^j)}(1+t^j)\frac{\partial p^j}{\partial g^j}\right] = L\frac{\partial h_s^j}{\partial p^j}\frac{\partial p^j}{\partial g^j}$$

Rearranging:

$$\begin{aligned}\frac{\partial N^j}{\partial g^j}h_d^j &= L\frac{\partial h_s^j}{\partial p^j}\frac{\partial p^j}{\partial g^j} - N^j\left[\frac{\partial h_d^j}{\partial g^j} + \frac{\partial h_d^j}{\partial p^j(1+t^j)}(1+t^j)\frac{\partial p^j}{\partial g^j}\right] \\ &= Lh_s^j\left(\frac{\partial h_s^j}{\partial p^j}\frac{p^j}{h_s^j}\right)\frac{\partial p^j}{\partial g^j}\frac{1}{p^j} - N^j\left[\frac{\partial h_d^j}{\partial g^j} + \frac{\partial h_d^j}{\partial p^j(1+t^j)}(1+t^j)\frac{\partial p^j}{\partial g^j}\right] \\ &= N^j h_d^j\frac{\theta^j}{p^j}\frac{\partial p^j}{\partial g^j} - N^j\left[\frac{\partial h_d^j}{\partial g^j} + \frac{\partial h_d^j}{\partial p^j(1+t^j)}(1+t^j)\frac{\partial p^j}{\partial g^j}\right]\end{aligned}$$

Dividing through by h_d^j:

$$\begin{aligned}\frac{\partial N^j}{\partial g^j} &= N^j\frac{\theta^j}{p^j}\frac{\partial p^j}{\partial g^j} - \frac{N^j}{h_d^j}\left[\frac{\partial h_d^j}{\partial g^j} + \frac{\partial h_d^j}{\partial p^j(1+t^j)}(1+t^j)\frac{\partial p^j}{\partial g^j}\right] \\ &= N^j\left[\frac{\theta^j}{p^j}\frac{\partial p^j}{\partial g^j} - \frac{\partial h_d^j}{\partial g^j}\frac{1}{h_d^j} - \frac{\partial h_d^j}{\partial p^j(1+t^j)}\frac{1+t^j}{h_d^j}\frac{\partial p^j}{\partial g^j}\right]\end{aligned}$$

$$
\begin{aligned}
&= N^j \left[\frac{\partial p^j}{\partial g^j} \frac{\theta^j}{p^j} - \frac{\partial p^j}{\partial g^j} \frac{\partial h_d^j}{\partial p^j (1+t^j)} \frac{1+t^j}{h_d^j} - \frac{\partial h_d^j}{\partial g^j} \frac{1}{h_d^j} \right] \\
&= N^j \left[\frac{\partial p^j}{\partial g^j} \frac{\theta^j}{p^j} - \frac{\partial p^j}{\partial g^j} \left(\frac{\partial h_d^j}{\partial p^j (1+t^j)} \frac{p^j (1+t^j)}{h_d^j} \right) \frac{1}{p^j} - \left(\frac{\partial h_d^j}{\partial g^j} \frac{g^j}{h_d^j} \right) \frac{1}{g^j} \right] \\
&= N^j \left[\frac{\partial p^j}{\partial g^j} \left(\frac{\theta^j - \eta^j}{p^j} \right) - \frac{\gamma^j}{g^j} \right] \qquad (18)
\end{aligned}
$$

The definitions of the elasticities are obvious from the substitutions.

This is equation (11b) in Epple-Zelenitz after sending J to infinity.

What this says is that more government spending must increase population in j if housing and the local public spending are substitutes ($\gamma < 0$). However, population in j could decrease if they are complements. In this case more government spending would tend to increase housing demand by existing residents. This would tend to bid up the price of housing. This would tend to block immigration and could even cause an outflow of population (logically possible, but surely unlikely).

9. Derivatives of $p_L^j(t^j, g^j)$.

   We apply the chain rule to (11) and use (4):

   (a) The derivative with the tax rate:

$$
\begin{aligned}
\frac{\partial p_L^j}{\partial t^j} &= \frac{\partial p_L^j}{\partial p^j} \frac{\partial p^j}{\partial t^j} \\
&= \frac{1}{L} \left\{ F[K_d^j(p^j), L] + p^j F_K {K'}_d^j - p_K {K'}_d^j \right\} \frac{\partial p^j}{\partial t^j} \\
&= \frac{F}{L} \frac{\partial p^j}{\partial t^j} \qquad (19)
\end{aligned}
$$

   (since $p^j F_K = p_K$).

   (b) We then have immediately:

$$
\frac{\partial p_L^j}{\partial g^j} = \frac{\partial p_L^j}{\partial p^j} \frac{\partial p^j}{\partial g^j} = \frac{F}{L} \frac{\partial p^j}{\partial g^j} \qquad (20)
$$

10. Equations (15)-(20) are at the foundation of everything in this literature. With some more work, you can derive the analogs for the small numbers case.

11. Epple-Zelenitz and the Objective of Profit Maximization.

To facilitate the comparison between maximizing π^j_{FS} and π^j_{NR}, we differentiate π^j_{FS} and only at the end set it equal to zero. Also, to ease notation, we omit the superscript j.

$$\begin{aligned}
\frac{\partial \pi_{FS}}{\partial t} &= L\left[ph_s + t\left(\frac{\partial p}{\partial t}h_s + p\frac{\partial h_s}{\partial p}\frac{\partial p}{\partial t}\right)\right] - \frac{\partial N}{\partial t}g \\
&= L\left[ph_s - t\frac{p}{1+t}\left(h_s + p\frac{\partial h_s}{\partial p}\right)\right] + \frac{N\theta}{1+t}g \\
&= L\left[ph_s - t\frac{p}{1+t}\left(h_s + h_s\theta\right)\right] + \frac{N\theta}{1+t}g \\
&= Lph_s\left[1 - \frac{t}{1+t}(1+\theta)\right] + \frac{N\theta}{1+t}g \\
&= \frac{N\theta}{1+t}\left\{g + Lph_s\left[\frac{1+t}{N\theta} - \frac{t}{N\theta}(1+\theta)\right]\right\} \\
&= \frac{N\theta}{1+t}\left[g + pLh_s\left(\frac{1-t\theta}{N\theta}\right)\right] \\
&= \frac{N\theta}{1+t}\left[g + pNh_d\left(\frac{1-t\theta}{N\theta}\right)\right] \\
&= \frac{N\theta}{1+t}\left[g + ph_d\left(\frac{1-t\theta}{\theta}\right)\right]
\end{aligned}$$

So at long last:

$$\frac{\partial \pi_{FS}}{\partial t} = 0 \implies tph_d = g + \frac{ph_d}{\theta}$$

The equality to the right of the arrow is equation (14a) in Epple-Zelenitz (at that point they have already sent J to infinity).

Their result for the derivative with g follows from the same procedure.

12. Henderson and the Objective of Net Revenue Maximization.

We have already done most of the work.

$$\begin{aligned}
\frac{\partial \pi_{NR}}{\partial t} &= \frac{\partial p_L}{\partial t} L + \frac{\partial \pi_{FS}}{\partial t} \\
&= \frac{\partial p^j}{\partial t^j} F + \frac{N\theta}{1+t}\left[g + ph_d\left(\frac{1-t\theta}{\theta}\right)\right] \\
&= -\frac{p^j}{1+t^j} F + \frac{N\theta}{1+t}\left[g + ph_d\left(\frac{1-t\theta}{\theta}\right)\right]
\end{aligned}$$

So:

$$\begin{aligned}
\frac{\partial \pi_{NR}}{\partial t} = 0 \implies & \frac{p^j}{1+t^j} F = \frac{N\theta}{1+t}\left[g + ph_d\left(\frac{1-t\theta}{\theta}\right)\right] \\
\implies & p^j F = N\theta\left[g + ph_d\left(\frac{1-t\theta}{\theta}\right)\right] \\
\implies & pNh_d = N\theta\left[g + ph_d\left(\frac{1-t\theta}{\theta}\right)\right] \\
\implies & \frac{ph_d}{\theta} = g + ph_d\left(\frac{1-t\theta}{\theta}\right) \\
\implies & \frac{ph_d}{\theta} = g + \frac{ph_d}{\theta} - tph_d \\
\implies & g = tph_d
\end{aligned}$$

The equality to the right of the arrow is equation (11) in Henderson (in per-capita form).

His result for the derivative with g follows from the same procedure.