Lecture 21

Wildasin (1991)

1. Overview

   (a) The agents in this model are "owners/rich" and "workers/poor."

   The workers/poor are costlessly mobile and paid their marginal product. The derive utility only from their own consumption.

   The owners/rich own factories (fixed factors) in their community and keep all income that is not paid to their workers. They derive utility from their own consumption and from consumption by a typical poor resident in their region (they are "altruistic").

   (b) The owners in each community choose to transfer resources from themselves to the workers in their own community (positive or negative). The analysis in the paper focuses on two kinds of Nash equilibria in this "redistributive transfer."

   In all cases, owners choose transfers taking into account that the population of workers is endogenous.

      i. Efforts by the rich to redistribute money to the poor have complicated effects. In-migration directly lowers wages earned by the workers, due to the diminishing marginal product of labor, but tends to increase rents received by owners. So there is a tradeoff.

      ii. Again, you can think of "worker migration" as part of the rules of the game. There is a migration function that is common knowledge with the property that, given variables in each region, the distribution of workers it implies leaves no individual worker with an incentive to migrate.

   (c) His key questions are:

      i. Is this kind of decentralized redistribution efficient?

      ii. If not, what kinds of instruments could a higher-tier government use to create an efficient allocation?

      iii. What are some interesting properties of the equilibria with higher-tier intervention?

   (d) What he leaves unanswered is:

      i. What set of local government instruments and property rights by owners would restore decentralized efficiency?

      ii. The higher-tier in this paper has as its objective to internalize fiscal externalities. What happens to the transfers if it has other objectives?

iii. When would a group of regions choose to be part of this federal system, if they could? On the one hand, the owners in "autarkic" regions do not have to worry about migration effects. On the other hand, the exogenous allocation of the workforce may be wildly inefficient (i.e., far from maximizing total output). What can we say about this tradeoff? This is the question analyzed in Rothstein-Hoover.

(e) NOTE: half of the mechanics in this paper are identical to those in Wildasin's capital mobility paper, with "labor" replacing "capital."

2. The Economy

   (a) There are n regions.

   (b) In each region there is a concave production function for "consumption good" depending on the number of workers in each region and region-specific fixed factors (suppressed).

   $$f_i(l_i)$$

   (c) There are two types of agents, worker/poor and owner/rich (the types are discussed somewhat more precisely in Rothstein-Hoover).

      i. Workers derive utility from total consumption:

      $$u(c_i)$$

      Without any loss of generality we represent the utility of a worker in region i by c_i.
      Workers inelastically supply a unit of labor in the same location where they consume.
      Workers may be moved among regions. The total number of workers in all regions must equal the total number available:

      $$\sum_{i=1}^{n} l_i = l$$

      ii. Owners derive utility from their own consumption and consumption by the poor in the region in which they reside. Thus:

      $$U_i(y_i, c_i)$$

      Owners cannot be moved among regions.

3. Efficient Allocations: The Central Planner Problem

   (a) At the most general level, the central planner problem is to choose $l_1, ..., l_n$, $y_1, ..., y_n$, and $c_1, ..., c_n$ to maximize $U_1(y_1, c_1)$ subject to utility constraints for all other agents (workers and owners), the aggregate resource constraint and total labor constraint.

(b) We will restrict attention to allocations in which all workers obtain the same utility. These are the only ones that could be decentralized, given free mobility. We therefore impose the restriction:

$$c_i = c, \quad i = 1, ..., n$$

The optimization problem is then:

$$\underset{y_1,...,y_n;l_1,...,l_n;c}{\text{Max}} \; U_1(y_1, c)$$

subject to:
$$U_i(y_i, c) = \bar{U}_i, \quad i = 2, ..., n$$
$$\sum_{i=1}^{n} f_i(l_i) = lc + \sum_{i=1}^{n} y_i$$
$$\sum_{i=1}^{n} l_i = l$$

Lagrangian:

$$\begin{aligned} \mathcal{L} &= U_1(y_1, c) \\ &+ \sum_{i=2}^{n} \mu_i [U_i(y_i, c) - \bar{U}_i] \\ &+ \phi[\sum_{i=1}^{n} f_i(l_i) - lc - \sum_{i=1}^{n} y_i] \\ &+ \lambda[\sum_{i=1}^{n} l_i - l] \end{aligned}$$

(c) Feel free to take the derivatives. When it is all done we obtain:

$$\sum_{i=1}^{n} \frac{\partial U_i / \partial c}{\partial U_i / \partial y_i} = l \tag{1}$$

$$f_1'(l_1) = ... = f_n'(l_n) \tag{2}$$

Equation (1) is a kind of "Samuelson condition." Improving the welfare of workers in one's own region improves their welfare in all regions and therefore increases the utility of owners in all other regions. Thus the common level of worker utility is like a public good for the owners in all regions.

Equation (2) says that the marginal product of labor must be equal in all regions. Thus, maximizing total product is necessary for efficiency in this model.

4. Incomes and Induced Preferences over Transfers $(z_1, ..., z_n)$.

   (a) Worker income.

   Workers in region i earn a wage in that region and may receive a transfer payment equal to z_i. Thus:

   $$c = w_i + z_i, \quad i = 1, ..., n$$

   (b) Firms.

   Firms within each community treat w_i and z_i as constant. They hire labor up to the point at which its marginal product equals the gross wage. Therefore in each community:

   $$f_i'(l_i) = w_i = c - z_i, \quad i = 1, ..., n$$

   This implicitly defines labor demand in region i as a function of the gross wage, $l_i(w_i)$.

   From the perspective of the local government, its choice of z_i changes the quantity of labor in the region by changing the wage that firms take as given when they choose a quantity of labor to hire. This relationship comes from inverting the previous equation:

   $$l_i(c - z_i), \quad i = 1, ..., n \tag{3}$$

   (c) Owner income.

   i. When no central government exists, owner income in region i is:

   $$y_i = f_i(l_i) - l_i f'(l_i) - z_i l_i, \quad i = 1, ..., n \tag{4}$$

   ii. When a central government exists, Wildasin assumes it provides a matching grant s_i to owners to help subsidize payments to the poor and levies a lump-sum tax T_i on the rich.

   Thus, with a central government, owner income in region i is:

   $$y_i = f_i(l_i) - l_i f'(l_i) - (1 - s_i) z_i l_i - T_i, \quad i = 1, ..., n \tag{5}$$

   In this case the central government faces a budget constraint:

   $$\sum_i (s_i z_i l_i - T_i) = 0$$

   (d) Induced Preferences over Transfers $(z_1, ..., z_n)$.

   In a "large numbers" model, the preferences of owners in region i over transfers would come from substituting (3) and (4) (or (5)) into $U_i(y_i, c)$.

   Owners would have no preferences over the transfers in other regions.

   As with the capital model, Wildasin focuses on the "small numbers" case.

   Owners have preferences over the transfers in other regions because the common level of consumption is affected by the transfers in every region. Thus, the amount of migration that changing z_i creates for region i depends on the level of transfers in every other region.

The formal derivation of these effects is *identical* to that in the capital model.

i. Substitute (3) back into the equation that defines it:

$$f_i'[l_i(c - z_i)] \equiv c - z_i$$

Differentiate both sides:

$$f_i'' l_i' = 1$$

Therefore:

$$l_i' = \frac{1}{f_i''} < 0$$

ii. Use (3) in the overall market clearing condition for labor (l is aggregate labor supply, assumed exogenous):

$$\sum_{i=1}^{n} l_i(c - z_i) = l$$

Use this to define the market clearing common level of consumption:

$$c(z_1, ..., z_n) \tag{6}$$

Substitute back in:

$$\sum_{i=1}^{n} l_i[c(z_1, ..., z_n) - z_i] = l$$

Differentiate both sides:

$$\frac{\partial c}{\partial z_i} = \frac{l_i'}{\sum_k l_k'} \equiv \sigma_i$$

Therefore:

$$0 > \sigma_i > 1$$

iii. The dependence of the quantity of labor in each community on transfers in each community is given by:

$$l_i(z_1, ..., z_n) \equiv l_i[c(z_1, \ldots, z_n) - z_i] \tag{7}$$

DO NOT CONFUSE (7) WITH (3).

Differentiate:

$$\frac{dl_i}{dz_i} = l_i' \left(\frac{\partial c}{\partial z_i} - 1 \right) = l_i'(\sigma_i - 1) > 0$$

The equilibrium worker population in j is:

$$l_j(z) \equiv l_j[c(z_1, \ldots, z_n) - z_j]$$

Therefore:

$$\frac{dl_j}{dz_i} = l_j' \left(\frac{\partial c}{\partial z_i} \right) = l_j' \sigma_i < 0$$

(e) Recall that direct owner utility is $U_i(y_i, c)$.

Induced preferences over transfers follow from using (6) to replace c and substituting (7) into (4) or (5) and using the result to replace y_i.

For example, substituting (17) into (5) gives:

$$y_i[l_i(z), z_i] \equiv f_i[l_i(z)] - l_i(z)f_i'[l_i(z)] - (1 - s_i)z_i l_i(z) - T_i$$

where z represents the vector $(z_1, ..., z_n)$. Using this and (6) gives:

$$U_i\{y_i[l_i(z), z_i], c(z)\} \tag{8}$$

This reduces to the induced preferences when there is no central government (so (4) holds) if we apply $s_i = T_i = 0$, all i.

5. The Equilibrium Problem

   Wildasin now sets up two one-shot games.

   (a) *Nash Equilibrium in Redistributive Transfers (NERT).*

   Owners in each region simultaneously choose redistributive transfers such that each owner is playing a best reply (according to (8) but with $s_i = T_i = 0$, all i). Payoffs to owners in region i depend on the migration that will result from different choices of z_i. Thus, playing a best reply means that owners anticipate the migration that will occur.

   (b) *Corrected Nash Equilibrium in Redistributive Transfers (CNERT).*

   The higher-tier government first chooses a vector of subsidies $(s_1, ..., s_n)$ to internalize the fiscal externalities. The exact way in which this is done is discussed below. Owners in each region then simultaneously choose redistributive transfers such that each owner is playing a best reply according to (8).[1]

   (c) His basic claim is that fiscal externalities imply that the NERT leads to an inefficient allocation while the corrected NERT leads to an efficient allocation. There is an efficiency rationale for a higher-tier government.

   What he actually shows is that there are spillovers with the NERT (Proposition 1) while (1) and (2) hold in a corrected NERT (Propositions 2 and 3).

6. In any Nash equilibrium we must have, for all i:

$$\frac{dU_i}{dz_i} = 0$$

[1] The vector of lump-sum transfers $(T_1, ..., T_n)$ is left undefined in this paper (there are n more unknowns than equations). Intuitively, the central government can use these to determine different distributions of owner utility across regions. Presumably there are classes of objective functions for the central government that would imply both a distribution of owner utility and internalization of the fiscal externalities, which is the behavior Wildasin assumes, but this is way outside the scope of this paper.

Therefore we need the derivatives of (8). Most of this work has already been done:

$$\begin{aligned}
\frac{dU_i}{dz_i} &= U_{iy}\frac{dy_i}{dz_i} + U_{ic}\frac{\partial c}{\partial z_i} \\
&= U_{iy}\left(\frac{\partial y_i}{\partial l_i}\frac{dl_i}{dz_i} + \frac{\partial y_i}{\partial z_i}\right) + U_{ic}\frac{\partial c}{\partial z_i} \\
&= U_{iy}\left\{[f_i' - f_i' - l_i f_i'' - (1-s_i)z_i]\frac{dl_i}{dz_i} - (1-s_i)l_i\right\} + U_{ic}\frac{\partial c}{\partial z_i} \\
&= U_{iy}\left\{-[l_i f_i'' + (1-s_i)z_i]\frac{dl_i}{dz_i} - (1-s_i)l_i\right\} + U_{ic}\frac{\partial c}{\partial z_i}
\end{aligned}$$

(a) The term in braces is the full effect of a change in transfer z_i on the income of the rich in i.

There are three components.

i. The first component is the change in $f_i - l_i f_i'$ from the induced change in l_i.
It is therefore the change in rents earned by fixed factors due to migration.

ii. The second component is the change in $(1-s_i)z_i l_i$ from the induced change in l_i.
It is therefore the change in total transfer payments due to payments to migrants.

iii. The third component is the direct change in $(1-s_i)z_i l_i$ from the change in z_i.
It is therefore the change in total transfer payments due to the change in the size of the payment to existing residents.

(b) Following Wildasin, divide both sides by U_{iy} and redefine the left hand side in terms of "real income" μ_i:

$$\frac{d\mu_i}{dz_i} = \frac{dU_i/dz_i}{U_{iy}} = -[l_i f_i'' + (1-s_i)z_i]\frac{dl_i}{dz_i} - (1-s_i)l_i + \text{MRS}_i\frac{\partial c}{\partial z_i}$$

Wildasin uses this expression because it is easier to interpret. That is the only reason!

(c) Use the expressions derived earlier to replace the derivatives on the right:

$$\frac{d\mu_i}{dz_i} = \text{MRS}_i\sigma_i - (1-s_i)l_i + [l_i f_i'' + (1-s_i)z_i]l_i'(1-\sigma_i)$$

Define:

$$\gamma_i \equiv \text{MRS}_i - l_i - (1-s_i)z_i l_i' \tag{9}$$

Then after a little algebra (use the fact $f_i'' l_i' = 1$):

$$\frac{d\mu_i}{dz_i} = \text{MRS}_i - (1 - s_i)l_i - (1 - \sigma_i)\gamma_i \tag{10}$$

Interpretation:

i. MRS_i is the marginal benefit of increasing z_i to the existing population.
ii. $(1 - s_i)l_i$ is the marginal cost of paying a higher benefit to the existing population.
iii. The last term takes into account all of the effects from the equilibrium migration response.

7. The analysis also takes into account the effect on region j of the choice may in i. If we write:

$$U_j \left\{ y_j[l_j(z), z_j], c(z) \right\}$$

then the derivative with $z_i \neq z_j$ is:

$$\begin{aligned} \frac{dU_j}{dz_i} &= U_{jy}\frac{dy_j}{dz_i} + U_{jc}\frac{\partial c}{\partial z_i} \\ &= U_{jy}\left(\frac{\partial y_j}{\partial l_j}\frac{dl_j}{dz_i}\right) + U_{jc}\frac{\partial c}{\partial z_i} \\ &= -U_{jy}[l_j f_j'' + (1 - s_j)z_j]\frac{dl_j}{dz_i} + U_{jc}\frac{\partial c}{\partial z_i} \end{aligned}$$

Again divide through by U_{jy}:

$$\frac{d\mu_j}{dz_i} = -[l_j f_j'' + (1 - s_j)z_j]\frac{dl_j}{dz_i} + \text{MRS}_j\frac{\partial c}{\partial z_i}$$

Again use the previous expressions to replace the derivatives:

$$\begin{aligned} \frac{d\mu_j}{dz_i} &= \frac{dy_j}{dz_i} + \text{MRS}_j\frac{\partial c}{\partial z_i} \\ &= \text{MRS}_j\sigma_i - [l_j f_j'' + (1 - s_j)z_j]l_j'\sigma_i \\ &= \sigma_i\gamma_j \end{aligned} \tag{11}$$

8. Proposition 1 ("spillovers exist in an uncorrected NERT").

   (a) In an uncorrected NERT,

   $$w_i \frac{d\mu_j}{dz_i} = -\epsilon_i l_i z_j > 0$$

   where $\epsilon_i \equiv (d\log l_i)/(d\log w_i)$. Note that $\epsilon_i < 0$ follows from the diminishing marginal product of labor (recall the derivation of (3) and the fact $l_i' = 1/f_i'' < 0$).

   So, in equilibrium, there would be a positive spillover to j from an increase in transfers in i.

   This spillover is larger the larger is the amount of transfer in j.

   (b) Proof.

   We are in a Nash equilibrium, so we have for all i:

   $$\frac{d\mu_i}{dz_i} = 0$$

   Therefore using (5):

   $$\text{MRS}_i = (1 - s_i)l_i + (1 - \sigma_i)\gamma_i$$

   Substitute this into the definition of γ_i (recall (9)):

   $$\gamma_i = (1 - s_i)l_i + (1 - \sigma_i)\gamma_i - l_i - (1 - s_i)z_i l_i'$$

   Solve for γ_i^* (Wildasin just calls it γ_i):

   $$\gamma_i^* = \frac{-s_i l_i - (1 - s_i)z_i l_i'}{\sigma_i}$$

   Since this is an uncorrected NERT, $s_i = 0$, giving:

   $$\gamma_i^* = -z_i l_i'/\sigma_i$$

   Now fix particular i and j. Since i was arbitrary above, we have:

   $$\gamma_j^* = -z_j l_j'/\sigma_j$$

   Therefore:

   $$\begin{aligned} \frac{d\mu_j}{dz_i} &= \sigma_i \gamma_j^* \\ &= -(\sigma_i/\sigma_j) l_j' z_j \\ &= -\left[\frac{l_i'}{\sum_k l_k'} \Big/ \frac{l_j'}{\sum_k l_k'}\right] l_j' z_j \\ &= -l_i' z_j \end{aligned}$$

Therefore:

$$w_i \frac{d\mu_j}{dz_i} = -w_i l_i' z_j = -\epsilon_i l_i z_j$$

which was to be shown.

9. Proposition 2 ("Equation (2) is satisfied in a corrected NERT")

   Wildasin does not set up an optimization problem for the central government. Instead, the central government's goal is to choose the vector of subsidies $(s_1, ..., s_n)$ so that the "marginal net external benefit" from the transfer in each region i is zero in the new equilibrium.

   He defines the marginal net external benefit to be:

$$\text{MEB}_i \equiv \sum_{j \neq i} \frac{d\mu_j}{dz_i} - \frac{d \sum_j s_j z_j l_j(z)}{dz_i}$$

   His assumption is that $\text{MEB}_i = 0$, $i = 1, ..., n$.

   What he shows in this proposition is that if $\text{MEB}_i = 0$, then the redistributive transfers are the same in each jurisdiction:

$$z_i = z_j, \quad \text{all } i, j$$

   It then follows that:

$$f_i'(l_i) = c - z_i = c - z_j = f_j'(l_j)$$

   so (2) holds.

   This means that he has the "right" concept of external benefit.

   (a) Proof. Using the previous results:

$$\begin{aligned}\frac{d \sum_j s_j z_j l_j(z)}{dz_i} = \sum_{j \neq i} \frac{d\mu_j}{dz_i} &= \sum_{j \neq i} \sigma_i \gamma_j^* \\ &= \sigma_i \sum_j \gamma_j^* - \sigma_i \gamma_i^* \\ &= \sigma_i \sum_j \gamma_j^* + s_i l_i + (1 - s_i) z_i l_i'\end{aligned}$$

   Now expand the left hand side:

$$\begin{aligned}\frac{d \sum_j s_j z_j l_j(z)}{dz_i} &= s_i l_i + \sum_j s_j z_j \frac{dl_j}{dz_i} \\ &= s_i l_i - s_i z_i l_i' + \sigma_i \sum_j s_j z_j l_j'\end{aligned}$$

Equating the two:

$$s_i l_i - s_i z_i l_i' + \sigma_i \sum_j s_j z_j l_j' = \sigma_i \sum_j \gamma_j^* + s_i l_i + (1 - s_i) z_i l_i'$$

Rearrange and clear:

$$\sum_j s_j z_j l_j' = \sum_j \gamma_j^* + (z_i l_i' / \sigma_i) = \sum_j \gamma_j^* + z_i \sum_j l_j'$$

Solve for z_i:

$$z_i = \frac{-\sum_j (\gamma_j^* - s_j z_j l_j')}{\sum_j l_j'}$$

The right hand side is the same for any i.

10. Proposition 3 ("Equation (1) is satisfied in a corrected NERT")