Lecture 20

1. Zodrow and Mieszkowski (1986), Part II
2. Wildasin (1986)

1. Zodrow and Mieszkowski, Part II.

   Underprovision of business public services

   (a) Individuals now derive utility from consumption C only.

   (b) "Output" (all purpose good) is produced by competitive firms within each jurisdiction using land, capital and now public infrastructure B:

   $$F(K, B)$$

   We have

   $$F_K > 0, \quad F_B > 0, \quad F_{KK} < 0, \quad F_{BB} < 0$$

   $$F_{KB} > 0$$

   (c) Output can be transformed (globally) into C and B in a 1:1 ratio. Therefore the overall resource constraint in each region is:

   $$B + C = F(K, B)$$

   Note that $MRT_{CB} = 1$.

   (d) Optimum

   Since all regions are identical, it is natural to restrict attention to optima in which all quantities (C, B, and K) are the same in all regions. Since there is a fixed capital stock we then necessarily have:

   $$K = \bar{K}/N$$

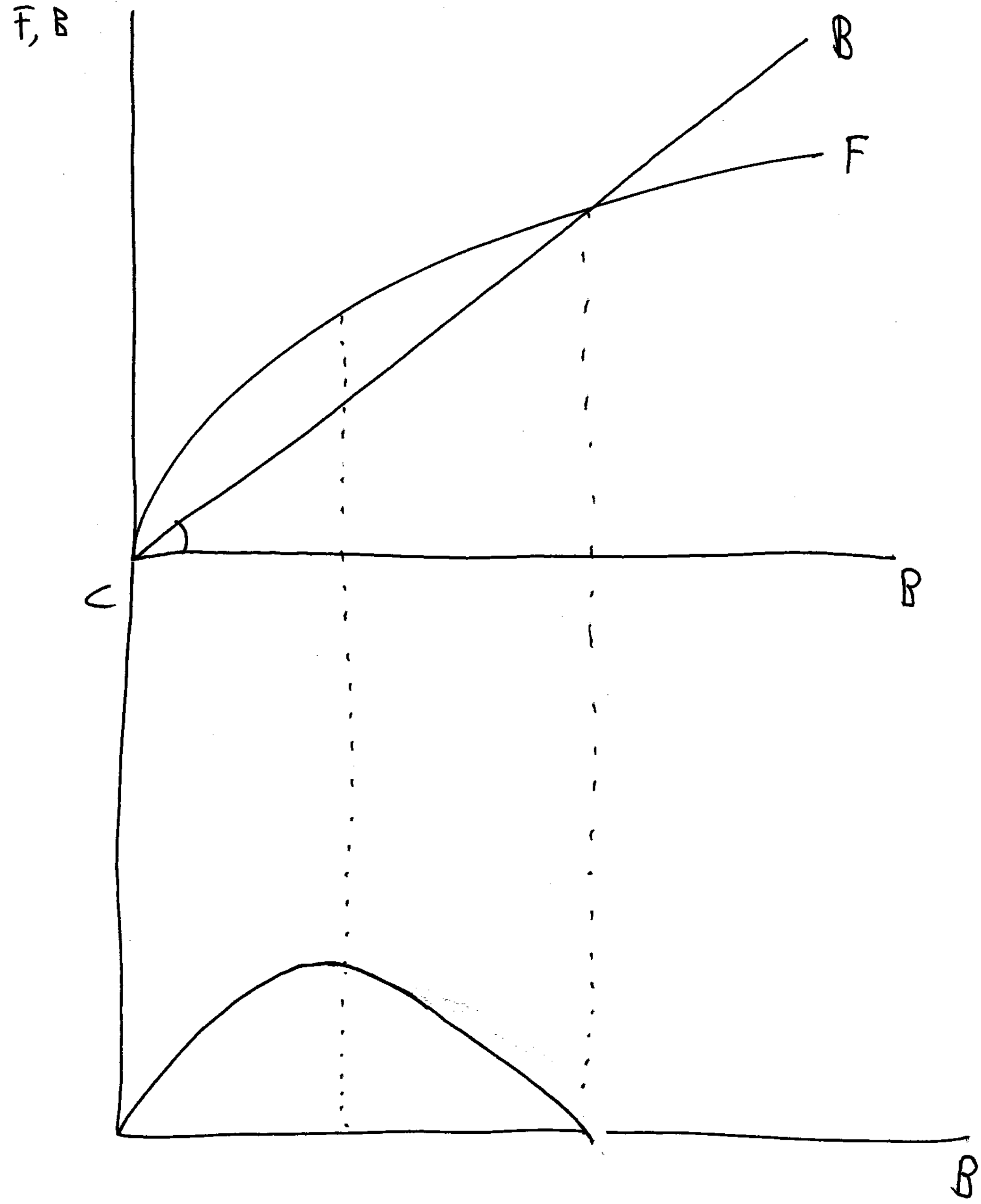
   The overall optimum problem can then be written:

   $$\max_B \; F(\bar{K}/N, B) - B$$

   This immediately gives for all regions:

   $$F_B = 1$$

Figure 1


F, B
B
F
C
B
B

(e) Equilibrium

i. Capital is perfectly mobile across jurisdictions. Thus, it must earn the same net return in every jurisdiction.
The *net* return is denoted r.
Producers treat r as exogenous.

ii. At the profit maximizing level of production, the quantity of capital employed is such that its marginal product equals its gross price:
$$r + T = F_K(K, B)$$
where T is a unit tax on capital.

iii. Local governments fund infrastructure with a unit tax on capital (T). The community's budget constraint is:
$$B = TK$$

iv. Each individual owns an equal share of the land in the jurisdiction of residence and an equal share of the national capital stock.
The individual budget constraint is therefore:
$$C = F(K, B) - (r + T)K + r(\bar{K}/N)$$
This is per-capita land rents plus the per-capita share of the total return to capital.

v. Local governments choose T to maximize consumption in the region. When they do this, they recognize that T affects the quantity of capital in the region in two ways. Recall $r + T = F_K(K, B)$. A change in T affects the quantity of capital directly by increasing the gross cost of capital in the region (left hand side); and indirectly by increasing the supply of infrastructure in the region ($B = TK$), which increases the marginal product of capital in the region (right hand side).
To find the derivative of K with T as recognized by local governments, substitute the budget constraint into the capital demand equation:
$$r + T = F_K(K, TK)$$
This defines $K(T)$. Substituting back in gives the identity:
$$r + T \equiv F_K[K(T), TK(T)]$$
Differentiating both sides gives:
$$1 = F_{KK}\frac{\mathrm{d}K}{\mathrm{d}T} + F_{KB}\left(K + T\frac{\mathrm{d}K}{\mathrm{d}T}\right)$$
This gives:
$$\phi \equiv -\frac{\mathrm{d}K}{\mathrm{d}T} = -\frac{1 - KF_{KB}}{F_{KK} + TF_{KB}}$$

vi. In theory, we could have $\frac{\mathrm{d}K}{\mathrm{d}T} > 0$, so an increase in the tax on capital drives in capital.

The reason is that the extra tax creates extra infrastructure which could increase the productivity of capital so much that the net return to capital increases.

Mieszkowski and Zodrow assume directly that this does not happen:[1]

$$\phi > 0$$

vii. We can now solve the optimization problem:

$$\max_{T} \; F[K(T), TK(T)] - (r+T)K(T) + r(\bar{K}/N)$$

This gives the first-order condition:

$$-F_K\phi + F_B(K - T\phi) - [K - (r+T)\phi] = 0$$

Rearranging gives:

$$F_B = \frac{K - (r+T)\phi + F_K\phi}{K - T\phi}$$

Using $r + T = F_K$ then gives:

$$F_B = \frac{K}{K - T\phi}$$

Therefore:

$$F_B = \frac{1}{1 - (T/K)\phi}$$

It follows from $T > 0$ and $\phi > 0$ that:

$$F_B > 1$$

Thus, the equilibrium is not efficient.

(f) We can be sure that there is less public infrastructure in equilibrium than in the optimum.

In both the equilibrium and the optimum, the allocation of capital across regions is the same. Thus then only reason F_B could differ in the two cases is that B differs in the two cases. Given that F is concave in B, we know that B is smaller in the equilibrium.

(g) Similar (but not identical) conclusions as in the previous part:

i. A marginal reduction in permitted head tax at the head tax optimum ($T = 0$) reduces public service.

ii. Technically, the latter result is not global: reductions in head tax (with T adjusting optimally) need not lead to monotonic reductions in public service. The situations in which this occurs are odd, however.

[1] They also assume that:

$$1 - KF_{KB} > 0$$

The marginal cost of diverting a unit of consumption to a unit of infrastructure, which is 1, exceeds the extra output associated with the higher marginal productivity of capital. This is their equation (16). This with $\phi > 0$ implies their equation (17). They do use (16) further on, but nowhere do they use (17).

2. Wildasin (1986)

   (a) This paper extends Zodrow and Mieszkowski (1986) in at least four ways.

      i. He gives a diagrammatic exposition of the inefficiency.

      ii. This is a "small numbers" model (except for the graph).
      Local tax policy affects the net return to capital in the economy. Since this is jointly determined by tax policy in every region, the tax policy of every other region plays a role in each region's choice of policy.

      iii. He provides a Pigouvian solution to the inefficiency: intergovernmental grants.

      iv. He conducts simulations to obtain a sense of how large the inefficiency is.

   (b) Illustration of the "externality."

      i. We have two communities, and both initially levy the same unit tax on capital $\bar{t}$.
      Capital is freely mobile, so it is distributed across the two regions so that the net return is equal.
      Community 1 is small, community 2 is big. Therefore we treat MP_2 as constant in the relevant range.
      Since capital is the only variable input, the area under the marginal product curve between 0 and K gives total *all purpose good* (or APG) available in that community.
      We use "APG" to emphasize that both private and public good are going to be funded from this total.

Figure 2

      ii. In this initial situation:

$$\begin{aligned} \text{Total APG} &= \text{APG in region 1 } + \text{ APG in region 2} \\ &= abK_1^*0 + F_2(\bar{K} - K_1^*) \end{aligned}$$

      Total APG is maximized since the marginal product of capital is equal across regions.

      iii. Now suppose region 1 increases its tax rate to t'.
      The quantity of capital in region 1 falls from K_1^* to K_1'.
      In this new situation:

$$\begin{aligned} \text{Total APG} &= \text{APG in region 1 } + \text{ APG in region 2} \\ &= aeK_1'0 + F_2[\bar{K} - K_1^* + (K_1^* - K_1')] \\ &= aeK_1'0 + F_2(\bar{K} - K_1^*) + gbK_1^*K_1' \end{aligned}$$

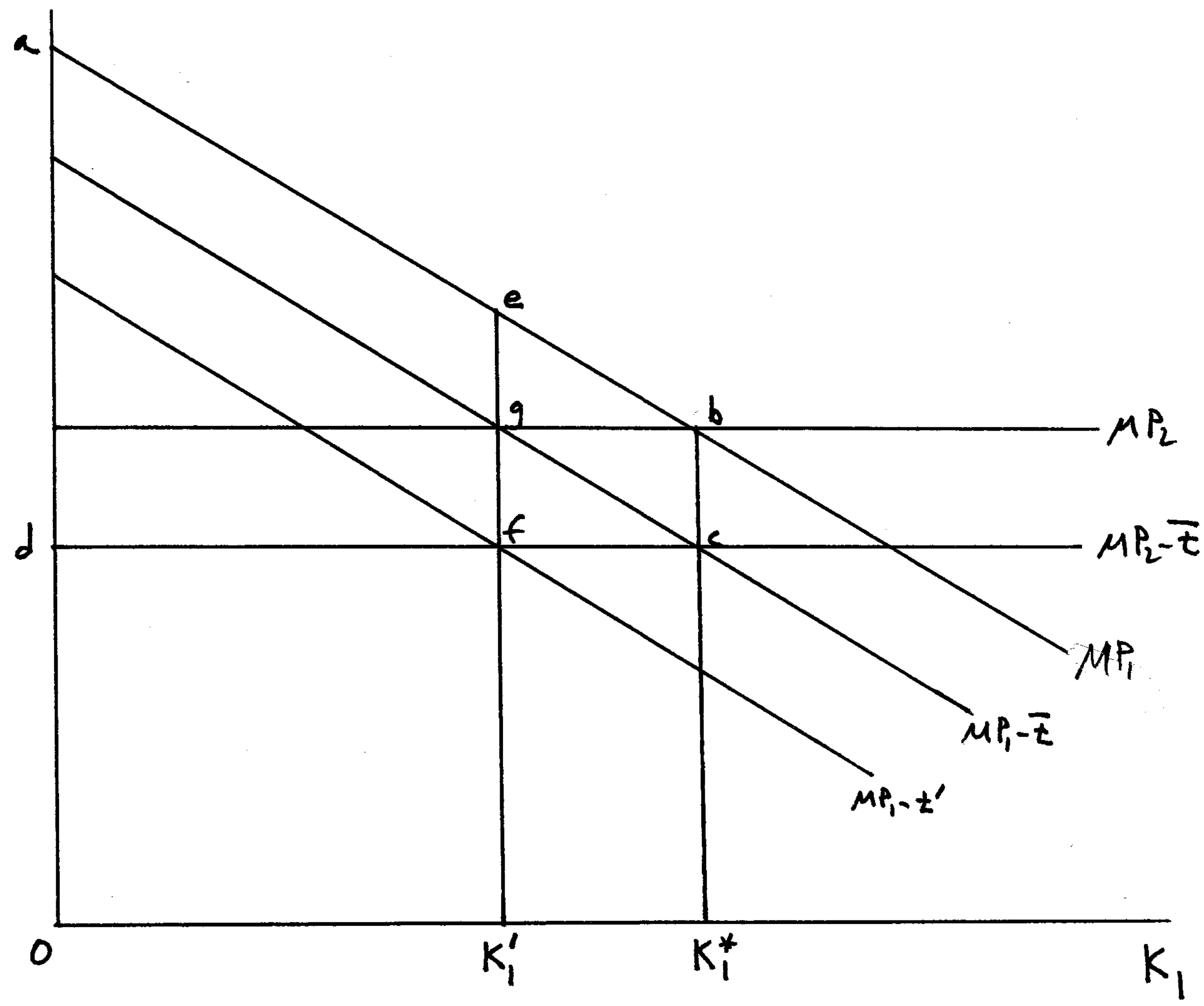


Figure 2

iv. The (absolute) change in total APG is therefore:

$$abK_1^*0 + F_2(\bar{K} - K_1^*) - aeK_1'0 - F_2(\bar{K} - K_1^*) - gbK_1^*K_1'$$
$$= (abK_1^*0 - aeK_1'0) - gbK_1^*K_1'$$
$$= ebK_1^*K_1' - gbK_1^*K_1'$$
$$= ebg$$

This is measure of the social loss due to the misallocation of capital across regions.

v. What is the loss as perceived by the sole resident of community 1 (equivalently, by the benevolent local government)?

Suppose the resident owns the fixed factor in the community but capital owners are absentee. This seems to be what Wildasin has in mind. The loss in capital causes a loss of APG produced in region 1 equal to $ebK_1^*K_1'$. The resident of region 1 was paying $fcK_1^*K_1'$ for capital, though. The private loss (equal to the net loss) is therefore:

$$ebK_1^*K_1' - fcK_1^*K_1' = ebcf$$

This overstates the true social loss. What is happening is that region 2 receives an increase in capital of $(K_1^* - K_1')$, and this generates APG there equal to $gbK_1^*K_1'$. Capital owners continue to receive $fcK_1^*K_1'$ and the resident/local government in region 2 receives the remainder, since $gbcf = \bar{t}(K_1^* - K_1')$ (recall, $\bar{t}$ is a unit tax). The loss to one government is a gain to the other.

vi. Since local loss exceeds social loss, and since the behavior of local governments is determined by local loss, the result is an inefficient allocation.

It is also natural to say that the out-flow of mobile capital creates a kind of externality. However, it is a pecuniary externality: capital moves in response to a reduction in the rate of return in a particular location. We do not usually think of pecuniary externalities as causing inefficiencies.

Perhaps one should think instead of capital substituting one region for another. Then we would have a kind of substitution effect, and we do usually think of the option to substitute as a source of inefficiency.

vii. As in Zodrow-Mieszkowski, production is efficient in a symmetric equilibrium to this game but the overall allocation is inefficient. All regions are identical, all tax rates are the same, so all have an equal share of capital. This equalizes the marginal product of capital across regions. Production of all purpose good is efficient. The problem is that the equilibrium level of the tax rate is too low.

viii. Finally, notice that if initially $\bar{t} = 0$ then the perceived loss from imposing the tax t' is ebg, which is also the social loss.

In contrast, with $\bar{t} > 0$, the resident in region 1 receives $gbcf$ before the tax increase and then loses it.

The initial situations differ by who has control over $gbcf$, the capital owners (if $\bar{t} = 0$) or the resident/local government ($\bar{t} > 0$). This creates the difference in the loss perceived by the local governments when the tax is increased.

(c) Wildasin asserts that the corrective subsidy should satisfy the marginal condition:

$$\frac{\mathrm{d}S_i}{\mathrm{d}t_i} = \sum_{j \neq i} t_j \frac{\mathrm{d}K_j}{\mathrm{d}t_i}$$

Intuitively, region i must recover APG equal to the part of its loss that is a transfer to other localities and not true social loss. This is the tax revenue received in those regions due to the inflow of capital.

i. A better way to proceed is to set up the planner's problem, derive the key necessary conditions for an optimum, and then show that this corrective subsidy leads to an equilibrium that satisfies the necessary conditions.

ii. He comes a little closer to doing this in the next paper we examine (Wildasin (1991)).

(d) To analyze this formula he needs to know more about the derivatives of the equilibrium capital stock:

$$\frac{\mathrm{d}K_i}{\mathrm{d}t_i}, \quad \frac{\mathrm{d}K_j}{\mathrm{d}t_i}$$

Wildasin uses these formulas in a lot his work, so let's go through them in some detail.

(e) Since capital is costlessly mobile the net return must be the same in every community. Let ρ denote the net return to capital.

Firms within each community treat ρ and t_i as constant. They hire capital up to the point at which its marginal product equals the gross price. Therefore in each community:

$$f_i'(K_i) = \rho + t_i, \quad i = 1, ..., n \tag{1}$$

Total capital demand must equal total capital supply:

$$\sum_j K_j = \bar{K} \tag{2}$$

We have $n + 1$ equations and $2n + 1$ variables.

i. One way to proceed is to appeal directly to the implicit function theorem. A vector of tax rates (and the total capital stock, $\bar{K}$, which is always suppressed) defines the equilibrium quantity of capital in each community and the equilibrium net return to capital:

$$K_i(t_1, ..., t_n), \;\; i = 1, ..., n \tag{3}$$

$$\rho(t_1, ..., t_n) \tag{4}$$

ii. We could then use the implicit function theorem to find the derivatives. The problem with this approach is that each derivative will be expressed in terms of derivatives of the production function. This is usually what one wants, since that is a primitive of the model, but in this case we happen to be more interested in the relationships the derivatives have *to each other*.
This is probably why Wildasin proceeds the way he does.

(f) Wildasin's procedure.

i. Return to the n equations defined in (1).
Apply the implicit function theorem to each equation separately to define the demand for capital in jurisdiction i:

$$K_i(\rho + t_i), \;\; i = 1, ..., n \tag{5}$$

DO NOT CONFUSE (5) WITH (3).
Substitute this back into the equation that defines it:

$$f_i'[K_i(\rho + t_i)] \equiv \rho + t_i$$

Differentiate both sides:

$$f_i'' K_i' = 1$$

Therefore:

$$K_i' = \frac{1}{f_i''} < 0$$

ii. Now substitute (5) into (2):

$$\sum_j K_j(\rho + t_j) = \bar{K}$$

This implicitly defines the equilibrium net return to capital, $\rho(t_1, ..., t_n)$.
Substituting that back into the equation that defines it gives:

$$\sum_j K_j[\rho(t_1, \ldots, t_n) + t_j] \equiv \bar{K}$$

Differentiate both sides:

$$K_1' \frac{\partial \rho}{\partial t_i} + \ldots + K_i' \left(\frac{\partial \rho}{\partial t_i} + 1 \right) + \ldots + K_n' \frac{\partial \rho}{\partial t_i} = 0$$

Therefore:

$$\frac{\partial \rho}{\partial t_i} = \frac{-K_i'}{\sum_j K_j'}$$

Notice that:

$$-1 < \frac{\partial \rho}{\partial t_i} < 0$$

iii. Finally, the equilibrium quantity of capital in each community is defined by:

$$K_i(t_1, ..., t_n) \equiv K_i[\rho(t_1, \ldots, t_n) + t_i]$$

Its derivatives are now easily derived and signed:

$$\frac{\partial K_i}{\partial t_i} = K_i' \left(\frac{\partial \rho}{\partial t_i} + 1 \right) < 0$$

$$\frac{\partial K_j}{\partial t_i} = K_j' \left(\frac{\partial \rho}{\partial t_i} \right) > 0$$

Notice that $\frac{\partial K_i}{\partial t_i} = K_i'$ if the derivative with ρ were zero, but it is not. Thus, the derivative of the equilibrium quantity of capital in i is not equal to the derivative of the (partial equilibrium) demand for capital in i.

(g) Wildasin then expresses these derivatives in terms of demand elasticities. Define (recall (5)):

$$\begin{aligned} \epsilon_j &\equiv \frac{\partial \log K_j}{\partial \log \rho + t_j} \\ &= K_j' \frac{(\rho + t_j)}{K_j} \end{aligned}$$

Therefore:

$$K_j' = \frac{\epsilon_j K_j}{\rho + t_j}$$

Then

$$\begin{aligned} \frac{\partial K_i}{\partial t_i} &= K_i' \left(1 + \frac{\partial \rho}{\partial t_i} \right) \\ &= \frac{\epsilon_i K_i}{\rho + t_i} \left(\frac{\sum_j K_j'}{\sum_j K_j'} - \frac{K_i'}{\sum_j K_j'} \right) \\ &= \frac{\epsilon_i K_i}{\rho + t_i} \left(\frac{\sum_{j \neq i} K_j'}{\sum_j K_j'} \right) \\ &= \frac{\epsilon_i K_i}{\rho + t_i} \left(\frac{\sum_{j \neq i} \frac{\epsilon_j K_j}{\rho + t_j}}{\sum_j \frac{\epsilon_j K_j}{\rho + t_j}} \right) \end{aligned}$$

This is his (9.1). His (9.2) follows similarly.