Lecture 18

Efficiency of Multi-Community Equilibrium with Mobile Labor: Introduction

1. It is very difficult to discuss the key papers in "the early literature" and a linear, logical analysis of the issues.

   By "the early literature" I mean Flatters, Henderson, Mieszkowski (1974), Hartwick (1980), and Boadway (1982). We have already discussed most of Hartwick (1980).

   The line of thought in this work more or less culminates with Myers (1990). We miss some interesting points by jumping straight to Myers, though. So, we will attempt to cut a brief and logical path through these papers.

2. Recall, first, the solution to the two-community optimum problem with a pure public good. The Lagrangian was:

$$
\begin{aligned}
\mathcal{L} &= U_1(G_1, X_1^i) \\
&+ \lambda[U_2(G_2, X_2^i) - \bar{U}_2] \\
&+ \mu[f_1(N_1) + f_2(N_2) - N_1X_1^i - N_2X_2^i - G_1 - G_2)] \\
&+ \psi[\bar{N} - N_1 - N_2]
\end{aligned}
$$

   Recall the notation:

$$F_i(N_i) = f_i'(N_i)$$

   We then derived the Samuelson condition:

$$N_i \frac{U_{i1}}{U_{i2}} = 1, \quad i = 1, 2$$

   and the locational efficiency condition:

$$F_1 - X_1 = F_2 - X_2$$

3. Flatters, Henderson, Mieszkowski (1974), to be referred to as FHM.

   They argue that migration equilibrium is not likely to satisfy the location efficiency condition. They argue that there is a place for a central (or federal) government to make redistributive transfers from one region to another, in order to achieve the proper allocation of the population.

   (a) First, note that $F_i - X_i$ is by definition the amount each individual sends to the government to pay for public goods. It is the excess of what the person is paid and what the person consumes.

   In general this could differ for each individual, but they (and we) assume everyone is treated identically. So since everyone is identical they earn the same wage and pay the same tax, which leaves them with the same consumption.

   (b) So, locational efficiency requires the per-capita tax payments to be the same across all regions. Will this hold in equilibrium?

   (c) Consider the following thought experiment.

   Suppose region "1" is identical to region "2." The budget constraint for a worker in region "1" is:

$$X_1 = F_1(N_1) - \tau_{1n}$$

   where τ_{1n} is a head tax. The government's budget constraint is:

$$N_1 \tau_{1n} = G_1$$

   Substituting this into the budget constraint gives:

$$X_1 + \frac{G_1}{N_1} = F_1(N_1)$$

   This is the solid line in Figure 1.[1]

Figure 1

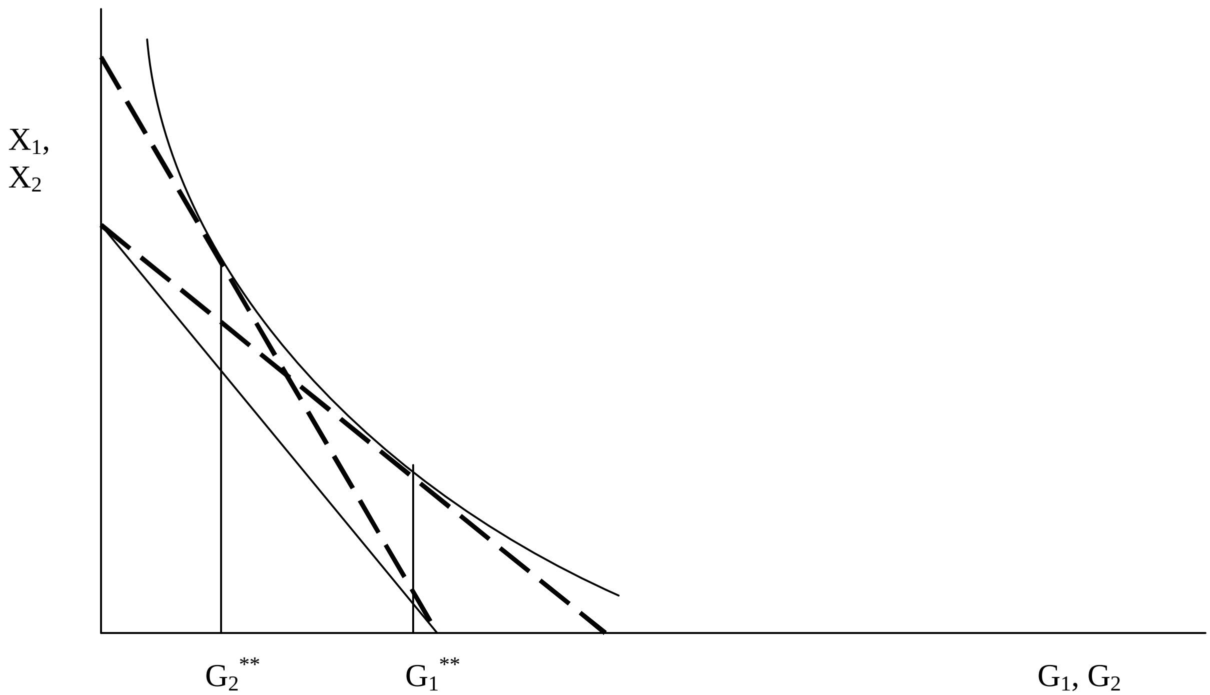
   Since both regions are identical and all agents are identical it is reasonable to assume the initial equilibrium is:

$$(G_1^*, X_1^*) = (G_2^*, X_2^*)$$

   (not drawn).

[1]If workers were the only agents or they owned the land in region 1 then the right hand side would be $f(N_1)/N_1$. Note the difference.

Figure 1



Now suppose we endow region "1" some additional fixed factor. Population flows from region "2" to region "1." Migration stops when utility is equalized again. This creates three effects:

i. The extra fixed factor in itself increases marginal product and shifts the constraint out:

$$\hat{F}_1(N_1) > F_1(N_1)$$

ii. The inflow of population shifts the curve back somewhat:

$$\hat{F}_1(N_1) > \hat{F}_1(\hat{N}_1)$$

iii. The inflow of population lowers the opportunity relative cost of the public good, so the slope becomes flatter:

$$\frac{1}{N_1} > \frac{1}{\hat{N}_1}$$

The new budget constraint in region 1 is:

$$X_1 + \frac{G_1}{\hat{N}_1} = \hat{F}_1(\hat{N}_1)$$

For simplicity (and to stay consistent with what FHM seem to intend) we assume that the first two effects, (i) and (ii), cancel each other. Thus, the new budget constraint is the same as the original but with a flatter slope. This is the flat dashed line in Figure 1.

Similarly, the budget constraint in region 2 is the same as the original but with a steeper slope.

$$X_2 + \frac{G_2}{\hat{N}_2} = \hat{F}_2(\hat{N}_2)$$

This is the steep dashed line in Figure 1.

Suppose at the new equilibrium we have (G_1^{**}, X_1^{**}) and (G_2^{**}, X_2^{**}).

(d) How does demand for the public good respond to these effects?

Suppose the price elasticity of demand is less than 1 (in absolute value). Then the price reduction in region 1 causes a representative resident to reduce total spending on the public good. Similarly, the price increase in region 2 causes a representative resident to increase total spending on the public good. As a result:

$$\frac{G_2^{**}}{\hat{N}_2} > \frac{G_2^{*}}{N_2} = \frac{G_1^{*}}{N_1} > \frac{G_1^{**}}{\hat{N}_1}$$

Going back to the budget constraints, we have:

$$\hat{F}_2(\hat{N}_2) - X_2^{**} = \frac{G_2^{**}}{\hat{N}_2} > \frac{G_1^{**}}{\hat{N}_1} = \hat{F}_1(\hat{N}_1) - X_1^{**}$$

We conclude that the location efficiency condition does not hold. Specifically, region 2 (the smaller region) is underpopulated and region 1 is overpopulated. FHM are clear on this point.

(e) Note that FHM refer to the *compensated* price elasticity of demand. The discussion above refers to the *uncompensated* price elasticity. Furthermore, the only reason the income elasticity was irrelevant is that I assumed away the income effect.

I invite someone to establish the FHM result under their single premise in a rigorous way. I doubt it can be done.

(f) FHM conclude by noting that a central government could restore efficiency by transferring resources from region 1 to region 2. Net of the transfers, individuals in both regions would make the same per-capita tax payment for public goods.

Again, FHM are clear on this point.

4. Boadway (1982)

(a) FHM never showed that the Samuelson condition would hold in equilibrium.

Boadway addresses this issue in essentially the same model. The only major difference is that instead of having two agents, "workers" and "landlords," Boadway has one, "worker-landlords."

These agents earn wages and are assumed to own an equal share of the land in both regions. Thus, they earn income from land rent in both regions and this income does not depend on where they reside.

This is a common assumption in this literature. Myers (1990) takes the same approach.

I somewhat prefer FHM's assumption. Alternatively, one could suppose that mobile agents lose their stake in region i when they leave and gain one in region j when they enter. Assuming everyone owns an equal share of the land in all regions is a bit extreme.

(b) Boadway also spends a good deal of time discussion "myopic" versus "non-myopic" governments.

We always assume governments anticipate migration – they are non-myopic. There isn't much interest in comparing myopia and non-myopia per se any more.

(c) As above, we have:

$$U_i(G_i, X_i), \quad f_i(N_i), \quad F_i = \frac{\partial f_i}{\partial N_i}$$

We also have rents in region i:

$$R_i(N_i) = f_i(N_i) - N_i F_i(N_i)$$

(d) Assume a head tax.

Then the budget constraint of an individual in region i is:

$$X_i + \frac{G_i}{N_i} = F_i(N_i) + \frac{R_i(N_i) + R_j(N_j)}{\bar{N}}$$

(e) The game.

This game is really a variation on the Cournot game. Recall that in the Cournot game, firms choose quantities and they anticipate the prices that will result by looking at the aggregate demand curve. "Consumers" are only present in spirit.

The same is true here. Regional government choose public goods and anticipate the population that will result. "Workers" are really only present in spirit.

Unlike the Cournot game, however, we are not just given a migration function. We have to derive it.

The migration equilibrium function has the following properties: given quantities of public goods in both regions, it specifies populations in both regions such that private good consumption is consistent with regional budget balance and no individual worker wants to migrate.

(f) Migration function.

From the individual budget constraint, define the function:

$$X_i(G_i, N_i, N_j) \equiv F_i(N_i) + \frac{R_i(N_i) + R_j(N_j)}{\bar{N}} - \frac{G_i}{N_i}$$

By assumption, individuals recognize that their private good consumption in region i depends on G_i and N_i, N_j in this way (and, the regional governments know this fact). This allows us to substitute this into the individual utility function to obtain:

$$V_i(G_i, N_i, N_j) \equiv U_i[G_i, X_i(G_i, N_i, N_j)]$$

We make the standard assumption that both communities are occupied in equilibrium. Migration equilibrium then requires:

$$\begin{aligned} V_i(G_1, N_1, N_2) &= V_2(G_2, N_2, N_1) \\ N_1 + N_2 &= \bar{N} \end{aligned}$$

This gives the migration functions:[2]

$$N_i(G_1, G_2) \;\; i = 1, 2$$

(g) Payoff functions.

The payoff function for region i is:

$$\mathcal{V}_1(G_1, G_2) \equiv V_1[G_1, N_1(G_1, G_2), N_2(G_1, G_2)]$$

$$\mathcal{V}_2(G_1, G_2) \equiv V_2[G_2, N_2(G_1, G_2), N_1(G_1, G_2)]$$

(h) Nash equilibrium.

We look for a Nash equilibrium in (G_1, G_2).

Treating G_{-i} as a constant, we maximize $\mathcal{V}_i(G_i, G_{-i})$. This gives us the reaction function for player i. Solving the two reaction functions simultaneously gives the Nash equilibrium.

(i) Solving for the reaction functions involves analyzing $N_i(G_1, G_2)$. While $N_i(\cdot)$ is intrinsically interesting, Boadway simply wants to show that the Samuelson condition holds in Nash equilibrium. If this is all you want, there is an easier way to proceed.

Assuming standard regularity conditions, we can also obtain the reaction function for region 1 by choosing $(G_1, X_1, X_2, N_1, N_2)$ to maximize $U_1(G_1, X_1)$ subject to the constraints used above. We have 5 choice variables and four multipliers, so nine unknowns; five FOCS's from the choice variables and four FOC's from the multipliers (which are just the constraints), so nine equations. Note that G_2 is treated as a constant.[3]

The Lagrangian is:

$$\begin{aligned} \mathcal{L} &= U(G_1, X_1) \\ &+ \lambda[U(G_1, X_1) - U(G_2, X_2)] \\ &+ \mu_1 \left[F_1(N_1) + \frac{R_1(N_1) + R_2(N_2)}{\bar{N}} - X_1 - \frac{G_1}{N_1} \right] \end{aligned}$$

[2]The literature tends to use expressions like $N_i(G_i, G_j)$, $i = 1, 2$, and similarly for the payoff functions. This notation should probably be avoided. The migration functions and payoff functions are defined over the same set of arguments. The order of the arguments should stay fixed. One can convey the same level of generality by writing $N_i(G_i, G_{-i}), i = 1, 2$, which is proper and unambiguous.

[3]You may object to this approach, since X_2, N_1, N_2 aren't really "chosen" by player 1. I remind you that in the standard utility maximization problem with two goods, a person doesn't really "choose" both goods. She chooses one good and "the constraint" chooses the other, at least as long as there is a unique way to satisfy the constraint. Yet, there are two first order conditions. The situation here is exactly analogous under the same uniqueness condition: there are five choice variables, but the individual really chooses just G_1 and the set of constraints determines X_1, X_2, N_1, N_2. Non-uniqueness raises both technical and fundamental issues (what values for the other variables should the individual expect when choosing G_1?). We leave those for another day.

$$+ \quad \mu_2 \left[F_2(N_2) + \frac{R_2(N_2) + R_1(N_1)}{\bar{N}} - X_2 - \frac{G_2}{N_2} \right]$$
$$+ \quad \psi[\bar{N} - N_1 - N_2]$$

Most of this is irrelevant, though! The first order condition with G_1 gives:

$$(1+\lambda)U_{11} = \frac{\mu_1}{N_1}$$

The first order condition with X_1 gives:

$$(1+\lambda)U_{12} = \mu_1$$

Therefore:

$$N_1 \frac{U_{11}}{U_{12}} = 1$$

The Samuelson condition holds.

(j) One can take the same approach to Boadway's analysis of the "property" tax (tax on land rent). Given Boadway's assumption about land ownership, and individual pays the same property tax regardless of where he or she resides. It will not distort the location decision. It may distort the choice within communities between private and public goods.

This is what Boadway shows: the Samuelson condition does *not* hold.

The structure is the same as above, except individuals receive income from the marginal product of labor and from an equal share of total land rent net of payments to pay for local public goods.

The tax is levied by each region on the rent earned by land within its jurisdiction. It is paid by owners regardless of where they reside. Thus, it is a source based tax.

Let τ_{ir} denote the property tax in region i. The budget constraint for a resident of region i is then:

$$X_i = F_i(N_i) + \frac{(1-\tau_{ir})R_i(N_i) + (1-\tau_{jr})R_j(N_j)}{\bar{N}}$$

Budget balance for each region requires:

$$\tau_{ir} R_i = G_i$$

Making the substitutions gives:

$$X_1 = F_1 + \frac{(R_1(N_1) - G_1)}{\bar{N}} + \frac{(R_2(N_2) - G_2)}{\bar{N}}$$

$$X_2 = F_2 + \frac{(R_2(N_2) - G_2)}{\bar{N}} + \frac{(R_1(N_1) - G_1)}{\bar{N}}$$

The Lagrangian is now:

$$\begin{aligned}
\mathcal{L} &= U(G_1, X_1) \\
&+ \lambda[U(G_1, X_1) - U(G_2, X_2)] \\
&+ \mu_1 \left[F_1 + \frac{(R_1(N_1) - G_1)}{\bar{N}} + \frac{(R_2(N_2) - G_2)}{\bar{N}} - X_1 \right] \\
&+ \mu_2 \left[F_2 + \frac{(R_2(N_2) - G_2)}{\bar{N}} + \frac{(R_1(N_1) - G_1)}{\bar{N}} - X_2 \right] \\
&+ \psi[\bar{N} - N_1 - N_2]
\end{aligned}$$

First order conditions with G_1 and X_1 give:

$$(1+\lambda)U_{11} = \frac{\mu_1 + \mu_2}{\bar{N}}$$

$$(1+\lambda)U_{12} = \mu_1$$

Result:

$$N_1 \frac{U_{11}}{U_{12}} = \frac{N_1}{\bar{N}} \frac{\mu_1 + \mu_2}{\mu_1}$$

Conclusion:

It is UNLIKELY that this is the Samuelson condition, although further analysis is required to be SURE that the right hand side is NOT in fact equal to 1!!

5. Neither the head tax alone nor the property tax alone seem to lead to an efficient decentralized equilibrium.

   FHM establish that the head tax alone is not enough. Boadway establishes the property tax alone is not enough.

   What if we have both?

6. The two-tax model.

   We develop this using the framework in Myers (1990). This is essentially the same as the model above, except the taxes are explicit and the notation is slightly different.

   (a) Gross wages are assumed equal to the marginal product of labor:

   $$w_i = F_i$$

All individuals own an equal share of each region's (and therefore of the nation's) land:

$$\left(\frac{T_1}{N}, \frac{T_2}{N}\right)$$

This gives each individual a claim on an equal share of each region's rents. Rents in region i are:

$$R_i = f_i - n_i F_i$$

The gross return per-unit of land is:

$$r_i \equiv \frac{R_i}{T_i}$$

Therefore, each individual receives *gross* income from land of:

$$r_1 \frac{T_1}{N} + r_2 \frac{T_2}{N} = \sum_k \left(\frac{R_k}{N}\right)$$

(b) Each region has available two taxes.

There is a head tax on residents:

$$\tau_{in}$$

There is also a *source based unit tax on land*. It is something like, "5 cents an acre." It is independent of the market value of the land. It must be paid by whoever owns the land (i.e., regardless of where the owner resides).

$$\tau_{ir}$$

Therefore, each individual receives *net* income from land of:

$$(r_1 - \tau_{1r})\frac{T_1}{N} + (r_2 - \tau_{2r})\frac{T_2}{N} = \sum_k \left(\frac{R_k}{N} - \tau_{kr}\frac{T_k}{N}\right)$$

(c) So, an individual in region i has the budget constraint:

$$x_i = w_i - \tau_{in} + \sum_k \left(\frac{R_k}{N} - \tau_{kr}\frac{T_k}{N}\right)$$

Assuming $C_i(n_i, Z_i) = Z_i$, the budget constraint for the government in region i is:

$$Z_i = \tau_{in} n_i + \tau_{ir} T_i$$

(d) An immediate and important implication is that the head taxes must be the same in both jurisdictions in any efficient equilibrium. This points seems to have been first made by Wildasin (1986).

From the budget constraint:

$$w_i - x_i = \tau_{in} - \sum_k \left(\frac{R_k}{N} - \tau_{kr} \frac{T_k}{N} \right)$$

In an efficient equilibrium, locational efficiency must hold. The left-hand side above is $F_i - x_i$. Thus, we must also have:

$$\tau_{1n} - \sum_k \left(\frac{R_k}{N} - \tau_{kr} \frac{T_k}{N} \right) = \tau_{2n} - \sum_k \left(\frac{R_k}{N} - \tau_{kr} \frac{T_k}{N} \right)$$

It follows that:

$$\tau_{1n} = \tau_{2n}$$

Head taxes must be the same in both locations.