Lecture 14

Tax Incidence: Two Sector Model

1. Recap of the definitions and results.

$$E = \left(\frac{X'}{X}\right)\frac{p_X}{p_Y}$$

$$\theta_{KX} = \frac{(p_K + T_{KX})K_X}{p_X X}$$

$$\theta_{LX} = \frac{p_L L_X}{p_X X}$$

$$S_X = \frac{d\log(K_X/L_X)}{d\log(f_K^X/f_L^X)}$$

$$S_Y = \frac{d\log(K_Y/L_Y)}{d\log(f_K^Y/f_L^Y)}$$

The final result (evaluated at $p_X = p_Y = p_K = p_L = 1$ and $T_{KX} = dp_L = 0$):

$$E(\theta_{KY} - \theta_{KX})dp_K + \theta_{LX}\frac{dL_X}{L_X} + \theta_{KX}\frac{dK_X}{K_X} = E\theta_{KX}dT_{KX}$$

$$S_Y dp_K - \frac{L_X}{L_Y}\frac{dL_X}{L_X} + \frac{K_X}{K_Y}\frac{dK_X}{K_X} = 0$$

$$-S_X dp_K - \frac{dL_X}{L_X} + \frac{dK_X}{K_X} = S_X dT_{KX}$$

In matrix form:

$$\begin{bmatrix} E(\theta_{KY} - \theta_{KX}) & \theta_{LX} & \theta_{KX} \\ S_Y & -\frac{L_X}{L_Y} & \frac{K_X}{K_Y} \\ -S_X & -1 & 1 \end{bmatrix} \begin{bmatrix} dp_K \\ \frac{dL_X}{L_X} \\ \frac{dK_X}{K_X} \end{bmatrix} = \begin{bmatrix} E\theta_{KX}dT_{KX} \\ 0 \\ S_X dT_{KX} \end{bmatrix}$$

$$\begin{aligned} dp_K &= \frac{1}{\Delta}\det\begin{bmatrix} E\theta_{KX}dT_{KX} & \theta_{LX} & \theta_{KX} \\ 0 & -\frac{L_X}{L_Y} & \frac{K_X}{K_Y} \\ S_X dT_{KX} & -1 & 1 \end{bmatrix} \\ &= \frac{E\theta_{KX}\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right) + S_X\left(\frac{\theta_{LX}K_X}{K_Y} + \frac{\theta_{KX}L_X}{L_Y}\right)}{E(\theta_{KY} - \theta_{KX})\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right) - S_Y - S_X\left(\frac{\theta_{LX}K_X}{K_Y} + \frac{\theta_{KX}L_X}{L_Y}\right)} dT_{KX} \end{aligned}$$

2. Tax incidence formulae.

   (a) Capital bears the tax if gross payments to capital as a share of national income are unchanged. Labor bears the tax if net payments to capital as a share of national income are unchnaged.

   Take these as definitions.

   (b) Capital bears the tax if:

$$\frac{d}{T_{KX}}\left(\frac{p_K K_0 + T_{KX} K_X}{p_K K_0 + p_L L_0 + T_{KX} K_X}\right) = 0$$

   Define:

$$I = p_K K_0 + p_L L_0 + T_{KX} K_X$$

   Differentiating both sides, using $T_{KX} = 0$ and the price normalizations (and staying with Tresch's style) gives:

$$\frac{dp_K K_0 + dT_{KX} K_X}{I} - \frac{K_0}{I^2}(dp_K K_0 + dT_{KX} K_X) = 0$$

   Rearrange:

$$dp_K K_0 \left[1 - \frac{K_0}{I}\right] = \frac{dT_{KX} K_X K_0}{I} - dT_{KX} K_X$$

   Simplify:

$$dp_K K_0 \left(\frac{L_0}{I}\right) = dT_{KX} K_X \left(\frac{K_0}{I} - 1\right)$$

   This gives the final result, which is a very famous expression:

$$dp_K = -\frac{dT_{KX} K_X}{K_X + K_Y}$$

   (c) Labor bears the tax if:

$$\frac{d}{T_{KX}}\left(\frac{p_K K_0}{p_K K_0 + p_L L_0 + T_{KX} K_X}\right) = 0$$

   Proceeding as above:

$$\frac{dp_K K_0}{I} - \frac{K_0}{I^2}(dp_K K_0 + dT_{KX} K_X) = 0$$

   Rearrange:

$$dp_K K_0 \left[1 - \frac{K_0}{I}\right] = \frac{dT_{KX} K_X K_0}{I}$$

   Simplify:

$$dp_K \left(\frac{L_0}{I}\right) = \frac{dT_{KX} K_X}{I}$$

This gives the final result:

$$dp_K = \frac{dT_{KX}K_X}{L_X + L_Y}$$

(d) Consistent with these results, we say capital bears most of the burden if:

$$dp_K < 0$$

and labor bears most of the tax if:

$$dp_K > 0$$

They share the burden equally if the price of capital relative to labor is unchanged:

$$dp_K = 0$$

3. We now give a close evaluation of the formula for dp_K.

(a) The denominator is positive.

We have S_X, S_Y and E all negative. The conclusion follows if the product:

$$(\theta_{KY} - \theta_{KX})\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right)$$

is negative.

By CRS:

$$\theta_{KX} + \theta_{LX} = 1 = \theta_{KY} + \theta_{LY}$$

Therefore:

$$\theta_{KY} - \theta_{KX} = \theta_{LX} - \theta_{LY}$$

So suppose:

$$\theta_{KY} - \theta_{KX} > 0$$

Then we also have:

$$\theta_{LX} - \theta_{LY} > 0$$

Using the price normalization, these imply, respectively:

$$\frac{K_Y}{Y} - \frac{K_X}{X} > 0, \quad \frac{L_X}{X} - \frac{L_Y}{Y} > 0$$

Rearranging gives:

$$\frac{K_X}{K_Y} < \frac{X}{Y}, \quad \frac{X}{Y} < \frac{L_X}{L_Y}$$

Therefore:

$$\frac{K_X}{K_Y} - \frac{L_X}{L_Y} < 0$$

which was to be shown.

(b) The previous analysis also shows that the concept of "factor intensity" in the model can be expressed in two equivalent ways. We have:

$$\theta_{KY} > \theta_{KX} \Longleftrightarrow \frac{K_X}{K_Y} < \frac{L_X}{L_Y} \Longleftrightarrow \frac{K_X}{L_X} < \frac{K_Y}{L_Y}$$

The first term says that the Y sector is capital intensive in the sense that capital costs are a greater share of total costs in the Y sector than they are in the X sector. The second term is the more traditional notion, that the capital/labor ration in Y exceeds that in X.

(c) The term with S_X is telling us something about factor substitution and the term with E is telling us something about demand for good X. We can think of these as a "substitution effect" and an "output effect."

i. The term for the substitution effect is always negative. This is intuitive. Raising the price of capital will cause substitution toward labor. This tends to bid down the net return to capital.

ii. Now consider the output effect:

$$E\theta_{KX}\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right)$$

Raising the price of capital in the X sector increases costs. Roughly speaking, this pushes the supply curve back. The X sector shrinks and the Y sector grows.

What this implies for dp_K depends on whether the X sector is capital intensive or labor intensive.

iii. If the X sector is capital intensive, then relatively large amounts of capital have to find employment in the Y sector. Intuitively, this should bid down the net return to capital. The output effect and the substitution effect then work the same way, and clearly $dp_K < 0$.

iv. If the X sector is labor intensive then the result is ambiguous. It is possible that labor bears most of the tax, i.e., $dp_K > 0$.

(d) If capital and labor are perfect substitutes in the Y sector, then capital and labor share equally the burden of taxation.

This is clear from the formula. If $S_Y = \infty$ then $dp_K = 0$.

(e) If both industries are equally factor intensive and each has the same elasticity of substitution between capital and labor, then capital bears the full burden of the tax.

Again, use the formula.

(f) See other results cited in Tresch.

4. A note on "Taxing the Demand Versus the Supply Side"

   Tresch writes:

   > Another implication of the perfectly elastic assumption is that it matters which side of the market is taxed.... Suppose the jurisdiction levies a tax on its own suppliers of capital through, say, a personal income tax. The citizens of the jurisdiction may reduce their saving...[but] they cannot possibly affect the given price of capital, p_K.... The citizens' after-tax return simply falls by the full amount of the tax, and they bear the entire burden of the tax (p. 567-568).

   What Tresch is describing is a *residence based* tax on capital. This means that the tax is levied by the jurisdiction where the owners reside on the total earnings of capital wherever it is deployed. This is in contrast to a *source based* tax on capital, which is how we usually assume the tax is levied. This means that the tax is levied by the jurisdiction where the capital is employed on the earnings within that jurisdiction. Tresch is *also* assuming that the owners of capital cannot move.

   Tresch's claim is obviously true: if the factor cannot escape the tax by being deployed in other jurisdictions and if the owners cannot escape the tax by moving then the owners must pay the tax. Weakening either assumption changes the result, though. The immobile owner of a factor that pays a source based tax can escape the tax by relocating the factor. The mobile owner of a factor that pays a residence based tax can escape the tax by relocating himself!