Lecture 13

Tax Incidence: Two Sector Model

1. Introduction

   (a) This model was pioneered by Arnold Harberger (1962, "The Incidence of the Corporation Income Tax," Journal of Political Economy (June)).

   His goal was to examine the effects of a tax on capital in one sector of an economy. He was interested in (1) how it affects the return to capital relative to labor and (2) the implications of this for "who" pays the tax.

   More specifically, he wanted to examine the implications of the idea that a tax on capital in one sector of an economy affects the relative net return to capital throughout the entire economy.

   This relative net return need not fall, since the intensity of use of capital in the taxed sector should matter. However, when it does fall, it is possible for income losses to capital to exceed the revenue gain to the government. The reason is that net capital income falls in both sectors while the government extracts revenue from just one sector. The tax creates some implicit income redistribution between capital and labor.

   (b) There are two perfectly competitive industries (sectors) which produce two goods in quantities X and Y under constant returns to scale.

   They use two factors of production, capital and labor, which are in fixed supply at K^0 and L^0.

   Factors are fully mobile between sectors and fully employed.

   The prices of the two goods are denoted p_X and p_Y, the wage rate by p_L, and the rate of return (rental price of capital) by p_K.

   (c) The particular tax is levied on the use of just one factor in just one sector (the "corporate" sector).

   This is called a "partial factor tax."

   (d) During the mid-1970's to the mid-1980's, the model was extended and used to analyze other kinds of taxes.

   One important extension was the incorporation of analytical techniques developed by Ron Jones (1965, "The Structure of Simple General Equilibrium Models," Journal of Political Economy (December)). Atkinson and Stiglitz (1980) follow this approach.

   Substantively, the extension of the model by Ballentine and Eris (1975, "On the General Equilibrium Analysis of Tax Incidence," Journal of Political Economy (June)) is especially important.

   See also the discussions of the literature in Myles and Tresch.

(e) As Atkinson and Stiglitz note, the assumption of fixed total amounts of labor and capital give the model a "short run" flavor, while the assumption that factors are fully mobile between sectors (especially labor) gives the model a "long run" flavor. Thus, the model is not clearly short-run or long-run.

"[The] assumptions are probably better seen as a useful analytical device, separating the different issues, than as corresponding to any actual time period."

2. Preliminaries

(a) Recall the one-sector partial equilibrium model. We have three equations and three unknowns:

$$X^D = X^D(p_X)$$

$$X^S = X^S(p_X)$$

$$X^D(p_X) = X^S(p_X)$$

The solution gives two equilibrium quantities and one equilibrium price: X_*^D, X_*^S, p_*^X.

(b) A *strictly* analogous two-sector general equilibrium model would have eighteen unknowns and eighteen equations.

Unknowns:

$$\begin{array}{llllll} X^D, & Y^D, & K_X^D, & L_X^D, & K_Y^D, & L_Y^D \\ X^S, & Y^S, & K_X^S, & L_X^S, & K_Y^S, & L_Y^S \\ p_X, & p_Y, & p_{KX}, & p_{KY} & p_{LX}, & p_{LY} \end{array}$$

Equations: Demand equations for all six items in the first row, supply equations for all six items in the second row, six equations equalizing demand and supply.

(c) The problem with this representation is that it misses two critical features of the two-sector model:

i. Factors are mobile across the sectors. There cannot be separate equilibrium prices for capital (like p_{KX} and p_{KY}) for each sector. Similarly for labor.

ii. The *aggregate* supply of capital and labor is assumed fixed.

To take these properties into account, we drop the concept of sector-specific factor supply, sector-specific market prices, and sector specific equilibrium. The system we actually analyze has twelve unknowns and twelve equations.

Unknowns:

$$\begin{array}{llllll} X^D, & Y^D, & K_X^D, & L_X^D, & K_Y^D, & L_Y^D \\ X^S, & Y^S, & & & & \\ p_X, & p_Y, & p_K, & & p_L & \end{array}$$

Equations: Demand equations for all six items in the first row, supply equations for all two items in the second row, two equations equalizing demand and supply, and the two equations:

$$L_X^D + L_Y^D = L^0$$

$$K_X^D + K_Y^D = K^0$$

The solution would give 8 equilibrium quantities and 4 equilibrium prices. All of these endogenous variables would be functions of the parameters of the system and any taxes. In principle, one can determine how any of the endogenous variables changes with a change in any of the taxes. One could use an appropriately general version of the implicit function theorem or differentiate the system equation-by-equation and gather terms.

(d) This is not the way Harberger proceeds, for two reasons:

   i. First, with CRS, price-taking firms do not have a well-defined profit maximization problem and well-defined factor demand curves. The analysis proceeds somewhat differently.

   ii. Harberger wants a solution with an interpretation, not just a formula. He therefore introduces demand elasticities and elasticities of substitution into the analysis.
   This inevitably leads to expressions in *ratios of quantities* and *shares*, not levels.

To develop this further, we need the following key result.

Theorem.
Suppose the production function in the X sector, $f^X(K_X, L_X)$, is constant returns to scale. Let p_K be the consumer price of capital (which is the net price here since the consumer must be the seller) and $p_K + T_{KX}$ the producer price (which is the gross price here since the producer must be the buyer). Then cost minimization and the requirement of zero profits in equilibrium imply:

$$p_K + T_{KX} = p_X f_K^X \tag{1}$$

$$p_L = p_X f_L^X \tag{2}$$

where:

$$f_K^X = \frac{\partial f^X}{\partial K_X}(K_X, L_X)$$

$$f_L^X = \frac{\partial f^X}{\partial L_X}(K_X, L_X)$$

Proof:

Under CRS, we always have:

$$X = f_K^X K_X + f_L^X L_X$$

so we always have:

$$p_X X = p_X f_K^X K_X + p_X f_L^X L_X$$

Cost minimization gives:

$$\frac{f_K^X}{f_L^X} = \frac{p_K + T_{KX}}{p_L}$$

These two equations give:

$$\frac{p_X X}{p_X f_K^X} = K_X + \frac{f_L^X}{f_K^X} L_X = K_X + L_X \frac{p_L}{p_K + T_{KX}}$$

Zero profits in the X industry means:

$$p_X X = (p_K + T_{KX})K_X + p_L L_X$$

so:

$$\frac{p_X X}{p_K + T_{KX}} = K_X + L_X \frac{p_L}{p_K + T_{KX}}$$

Equating gives:

$$\frac{p_X X}{p_X f_K^X} = \frac{p_X X}{p_K + T_{KX}}$$

The denominators must be equal, which gives the result.

(e) We now have the system:

$$p_K + T_{KX} = p_X f_K^X(K_X^D, L_X^D)$$

$$p_L = p_X f_L^X(K_X^D, L_X^D)$$

$$p_K = p_Y f_K^Y(K_Y^D, L_Y^D)$$

$$p_L = p_Y f_L^Y(K_Y^D, L_Y^D)$$

$$X^D = X^D(\cdot)$$

$$X^S = f^X(K_X^D, L_X^D)$$

$$X^D(\cdot) = f^X(K_X^D, L_X^D)$$

$$Y^D = Y^D(\cdot)$$

$$Y^S = f^Y(K_Y^D, L_Y^D)$$

$$Y^D(\cdot) = f^Y(K_Y^D, L_Y^D)$$

$$L_X^D + L_Y^D = L^0$$

$$K_X^D + K_Y^D = K^0$$

This system has 12 equations and 12 unknowns, determining 8 quantities and 4 prices.

One equation is redundant because consumer demands must satisfy the budget constraint. By CRS:

$$p_X X = p_L L_X + p_K K_X$$

$$p_Y Y = p_L L_Y + p_K K_Y$$

Adding up gives:

$$p_L(L_X^D + L_Y^D) + p_K(K_X^D + K_Y^D) = p_X X + P_Y Y = p_L L^0 + p_K K^0$$

It now follows that $L_X^D + L_Y^D = L^0$ implies $K_X^D + K_Y^D = K^0$.

(f) Harberger makes the price of labor the numeraire, so $p_L = 1$ identically.

He drops $Y^D = Y^D(\cdot)$ from the system.

He does not use superscripts to distinguish demand and supply. It should be clear from the context.

(g) The analysis now proceeds by assuming we are at an equilibrium and the system has been solved for all endogenous quantities as a function of T_{KX}.

By assumption, the quantities are scaled so that at the solution all prices are equal to 1. For example, if one unit of capital costs 2, two units of "half-capitals" would each cost 1. He does this so that:

$$p_K(1 + T_{KX}) = p_K + T_{KX}$$

This means that T_{KX} can be thought of as both a unit tax and an ad valorem tax.

(h) Note that you should not replace any prices with "1" in any expression you are going to differentiate. You do not want to confuse a function with that function evaluated at a particular point.

(i) He then differentiates the system with respect to T_{KX}. This generates differential terms for the endogenous variables.

The purpose of the manipulations is to generate an expression in which only two differential terms appear, dp_K **and** dT_{XY}.

3. The Goods-Demand Equations

   (a) On the demand side, aggregate demands are generated by the maximization of a single utility function subject to an aggregate budget constraint. The utility function is:

   $$U(X, Y)$$

   The budget constraint recognizes that the single individual owns all the labor and capital in the economy and also owns the industries in both sectors. With constant returns to scale there are no profits, however, so ownership of the industries generates no return per se.

   $$p_X X + p_Y Y = p_L L_0 + p_K K_0$$

   In general this would give:

   $$X = X^D(p_X, p_Y, p_L, p_K)$$

   $$Y = Y^D(p_X, p_Y, p_L, p_K)$$

   (b) In fact, Harberger writes:

   $$X = X^D(p_X/p_Y)$$

   The dependence on just the price ratio means utility is homothetic.

   The absence of the income terms is trickier. Harberger's assumptions are intended to imply that this is unaffected by the tax. It is therefore a constant and just be suppressed.

   Harberger assumes that the government levies an infinitesimal tax in an economy without any distortions. So, there is no excess burden. Second, the tax revenue is returned lump-sum to the individual. Thus, the tax has no direct effect on incomes.

   Since extra money is raised and spent, this is "balanced-budget incidence."

   (c) "Totally" differentiating gives:

   $$dX = (X')\frac{p_Y dp_X - p_X dp_Y}{p_Y^2}$$

   Using the price normalization gives:

   $$\frac{dX}{X} = \left(\frac{X'}{X}\right)(dp_X - dp_Y)$$

   Define:

   $$E = \left(\frac{X'}{X}\right)\frac{p_X}{p_Y} = \left(\frac{X'}{X}\right)$$

This gives:

$$\frac{dX}{X} = E(dp_X - dp_Y)$$

4. Goods-Supply

   Totally differentiating the production function gives:

$$dX = f_K^X dK_X + f_L^X dL_X$$

$$\frac{dX}{X} = \frac{f_K^X K_X}{X}\frac{dK_X}{K_X} + \frac{f_L^X L_X}{X}\frac{dL_X}{L_X} = \frac{p_X f_K^X K_X}{p_X X}\frac{dK_X}{K_X} + \frac{p_X f_L^X L_X}{p_X X}\frac{dL_X}{L_X}$$

   Using (1) and (2):

$$\frac{dX}{X} = \frac{(p_K + T_{KX})K_X}{p_X X}\frac{dK_X}{K_X} + \frac{p_L L_X}{p_X X}\frac{dL_X}{L_X}$$

   Define:

$$\theta_{KX} = \frac{(p_K + T_{KX})K_X}{p_X X}$$

$$\theta_{LX} = \frac{p_L L_X}{p_X X}$$

   Then:

$$\frac{dX}{X} = \theta_{KX}\frac{dK_X}{K_X} + \theta_{LX}\frac{dL_X}{L_X}$$

5. Equating the percentage change in goods demand and goods supply gives an equation that *links changes in factor demands to changes in goods prices*:

$$E(dp_X - dp_Y) = \theta_{KX}\frac{dK_X}{K_X} + \theta_{LX}\frac{dL_X}{L_X} \tag{3}$$

6. Turning to input demands, we use (1) and (2) as ratios:

$$\frac{f_K^X}{f_L^X} = \frac{p_K + T_{KX}}{p_L}$$

This is an identity at the solution, meaning quantities and prices will adjust to a change in T_{KX} to preserve the equality. We can therefore differentiate both sides:

$$d\log\left(\frac{f_K^X}{f_L^X}\right) = d\log\left(\frac{p_K + T_{KX}}{p_L}\right)$$

and

$$d\log\left(\frac{f_K^Y}{f_L^Y}\right) = d\log\left(\frac{p_K}{p_L}\right)$$

We have:

$$\begin{aligned} d\log\left(\frac{p_K + T_{KX}}{p_L}\right) &= d\left[\log(p_K + T_{KX}) - \log(p_L)\right] \\ &= d\log(p_K + T_{KX}) - d\log(p_L) \\ &= d(p_K + T_{KX})/(p_K + T_{KX}) - dp_L/p_L \\ &= dp_K + dT_{KX} - dp_L \end{aligned}$$

using $p_K = 1$ at the initial equilibrium and $T_{KX} = 0$ and $p_L = 1$ always. Similarly:

$$d\log\left(\frac{p_K}{p_L}\right) = d\log(p_K) - d\log(p_L) = dp_K - dp_L$$

Tresch defines the elasticity of substitution as the percentage change in the input ratio with respect to the change in the rate of technical substitution:

$$S_X = \frac{d\log(K_X/L_X)}{d\log(f_K^X/f_L^X)}, \quad S_Y = \frac{d\log(K_Y/L_Y)}{d\log(f_K^Y/f_L^Y)}$$

Using all of the previous results gives:

$$\frac{dK_X}{K_X} - \frac{dL_X}{L_X} = S_X(dp_K + dT_{KX} - dp_L)$$

$$\frac{dK_Y}{K_Y} - \frac{dL_Y}{L_Y} = S_Y(dp_K - dp_L)$$

We can simplify these expressions using $dp_L = 0$, $dK_Y = -dK_X$, and $dL_Y = -dL_X$. The final result is two equations that *link changes in factor demands to changes in factor prices*:

$$\frac{dK_X}{K_X} - \frac{dL_X}{L_X} = S_X(dp_K + dT_{KX}) \tag{4}$$

$$-\frac{dK_X}{K_Y} + \frac{dL_X}{L_Y} = S_Y dp_K \tag{5}$$

7. The last step is to derive two equations that link changes in goods prices to changes in factor prices. This will allow us to remove dp_X and dp_Y from the system.

   A natural place to start is with a relationship that links p_X with p_K and p_L. The only obvious candidate is:

$$p_X X = p_L L_X + (p_K + T_{KX})K_X$$

   Differentiating both sides:

$$p_X dX + X dp_X = p_L dL_X + L_X dp_L + (p_K + T_{KX})dK_X + (dp_K + dT_{KX})K_X \tag{6}$$

   We need to eliminate dX. From the production function:

$$dX = f_L^X dL_X + f_K^X dK_X$$

   Multiply through by p_X and use (1) and (2) to eliminate the marginal products. This gives:

$$p_X dX = p_L dL_X + (p_K + T_{KX})dK_X$$

   Using this in (6) reduces it to:

$$X dp_X = L_X dp_L + (dp_K + dT_{KX})K_X$$

   Divide through by X:

$$dp_X = \frac{L_X}{X} dp_L + \frac{K_X}{X}(dp_K + dT_{KX})$$

   This is equivalent to:

$$dp_X = \theta_{LX} dp_L + \theta_{KX}(dp_K + dT_{KX})$$

Repeating the analysis for the Y industry gives:

$$dp_Y = \theta_{LY} dp_L + \theta_{KY} dp_K$$

As before, using the fact $dp_L = 0$, these become:

$$dp_X = \theta_{KX}(dp_K + dT_{KX}) \tag{7}$$

Repeating the analysis for the Y industry and using $dP_L = 0$ gives:

$$dp_Y = \theta_{KY} dp_K \tag{8}$$

Note that θ_{LX} and θ_{LY} do not appear.

8. The Final Expression

The key equations are (3)-(5) and (7),(8).

Use (7),(8) in (3) to remove dp_X and dp_Y:

$$E[\theta_{KX}(dp_K + dT_{KX}) - \theta_{KY} dp_K] = \theta_{KX} \frac{dK_X}{K_X} + \theta_{LX} \frac{dL_X}{L_X}$$

Rearranging gives the expression in Tresch (except his T_{KX} should really be dT_{KX}, and his footnote 19 that attempts to explain his choice is not correct):

$$E(\theta_{KY} - \theta_{KX}) dp_K + \theta_{LX} \frac{dL_X}{L_X} + \theta_{KX} \frac{dK_X}{K_X} = E\theta_{KX} dT_{KX} \tag{9}$$

Rearranging equation (5) gives:

$$S_Y dp_K - \frac{L_X}{L_Y} \frac{dL_X}{L_X} + \frac{K_X}{K_Y} \frac{dK_X}{K_X} = 0 \tag{10}$$

Equation (4) gives:

$$-S_X dp_K - \frac{dL_X}{L_X} + \frac{dK_X}{K_X} = S_X dT_{KX} \tag{11}$$

The point of all this becomes somewhat clearer if we write this in matrix form:

$$\begin{bmatrix} E(\theta_{KY}-\theta_{KX}) & \theta_{LX} & \theta_{KX} \\ S_Y & -\frac{L_X}{L_Y} & \frac{K_X}{K_Y} \\ -S_X & -1 & 1 \end{bmatrix} \begin{bmatrix} dp_K \\ \frac{dL_X}{L_X} \\ \frac{dK_X}{K_X} \end{bmatrix} = \begin{bmatrix} E\theta_{KX} dT_{KX} \\ 0 \\ S_X dT_{KX} \end{bmatrix}$$

A straightforward application of Cramer's Rule gives the result we want:

$$\begin{aligned} dp_K &= \frac{1}{\Delta}\det\begin{bmatrix} E\theta_{KX} dT_{KX} & \theta_{LX} & \theta_{KX} \\ 0 & -\frac{L_X}{L_Y} & \frac{K_X}{K_Y} \\ S_X dT_{KX} & -1 & 1 \end{bmatrix} \\ &= \frac{E\theta_{KX}\left(\frac{K_X}{K_Y}-\frac{L_X}{L_Y}\right)+S_X\left(\frac{\theta_{LX}K_X}{K_Y}+\frac{\theta_{KX}L_X}{L_Y}\right)}{E(\theta_{KY}-\theta_{KX})\left(\frac{K_X}{K_Y}-\frac{L_X}{L_Y}\right)-S_Y-S_X\left(\frac{\theta_{LX}K_X}{K_Y}+\frac{\theta_{KX}L_X}{L_Y}\right)} dT_{KX} \end{aligned}$$