**Lecture 12**

Outline

1. Overview of tax incidence analysis
2. The representation of taxation
3. Incidence in a partial equilibrium model (Kotlikoff and Summers)
4. Taxation and oligopoly

1. Overview of tax incidence analysis
   (a) "The study of tax incidence is, broadly defined, the study of the effects of tax policies on the distribution of economic welfare. It bridges both the positive and normative aspects of public economics." (Kotlikoff and Summers)
   (b) More precisely, tax incidence studies typically produce one of two kinds of output.
      i. Predictions about price changes.
         These are very "positive" in flavor.
         However, when prices are wage rates and interest rates, the "normative" is there in the background.
      ii. Predictions about changes in the distribution of income or the degree of income inequality. That is to say, the impact on GROUPS.
         One obtains results like, "20% of the tax is paid by the richest 5%."
         These studies usually use results from "price change" studies to derive their conclusion. In other words, it is assumed that particular taxes are shifted to some specified degree. The implications of this for the distribution of income or income inequality are then traced out in what is more or less an accounting exercise.
         These measures obviously exclude excess burden in the measure of incidence.
      iii. Neither of these studies fully accounts for the distribution of the "total burden" of a tax.
         To do this, one could first define individual "total burden" using (say) the (negative of the) equivalent variation for the change from the no-tax situation to the post-tax situation.
         One could then aggregate over groups as desired to report a distribution of total burden across groups.
         Computable general equilibrium studies sometimes do this.

(c) Incidence studies are often concerned with the difference between the seeming intent of a policy and its outcome.

  i. The distribution of economic welfare that results in the post-tax equilibrium depends on ALL of the determinants of the new equilibrium. Policy makers do choose SOME of these determinants, like the magnitude of the tax rate and the tax base.
  However, these interact with other determinants that are outside their control to limit what policy can accomplish.

  ii. Furthermore, policy makers also control variables that are NOT determinants of the new equilibrium, like the legal incidence of the tax. Controlling these has no effect on incidence.

(d) The interest in groups and in unintended consequences suggests that the motivation for an interest in incidence is precisely the recognition that there is something wrong with the optimal tax paradigm.

  If taxes are social welfare maximizing (and regardless of whether they are first-best or second-best), then incidence is not fundamentally interesting to the social-planner/economist.

  Incidence is a detail in the solution to the OT problem, and there is nothing for the planner/analyst to do with the information.

  Perhaps the needed social welfare function doesn't exist, or it exists but maximizing it requires institutions (of redistribution) that do not exist.

(e) Most studies use one of two analytical methods.

  i. Develop a simple analytical model and take derivatives.
  The information about incidence comes from examining the signs of derivatives, often evaluated at an initial zero-tax state.

  ii. Computable general equilibrium models.
  The information about incidence comes from examining pages of numbers.

(f) Finally, when the study is "general equilibrium" in nature, one generally specifies what happens to the tax revenue.

  i. Balanced-Budget Incidence.
  EXTRA money will be raised and the same amount spent.
  Incidence must take into account how the money is spent. However, sometimes the expenditure is assumed to be lump-sum return of revenue.

ii. Differential Incidence.
The SAME money is raised, but you substitute one tax for another. Again, sometimes one of the taxes is assumed to be the lump-sum tax.

2. The representation of taxation

   (Brief review of notes in the course reader).

3. Incidence in a partial equilibrium model

   (a) The general analytical method is as follows.

      i. The price system can be characterized by gross and net prices. We do not need to use consumer and producer prices.
      This is because the identity of buyers and sellers in every market is fixed.
      ii. Taxes enter the system as exogenous variables.
      iii. Equilibrium prices are determined by the set of market clearing conditions as functions of the things that are exogenous: preferences, technology, aggregate resources, and now taxes.
      iv. Thus, a tax in one market is a determinant of the equilibrium prices in all markets.
      This point is sometimes forgotten, since partial equilibrium analysis is so common and it usually considers just one market.
      v. To study incidence, we substitute these "market-clearing-price functions" back into the set of equilibrium conditions and differentiate with respect to the tax rate or rates.

   (b) To illustrate, consider a single-market partial equilibrium model.
   Demand Curve:

   $$X_D(p_N + t)$$

   Supply Curve:

   $$X_S(p_N)$$

   Equilibrium:

   $$X_D(p_N + t) = X_S(p_N)$$

   This defines the net price as a function of the tax rate:

   $$p_N(t)$$

   Substituting back in gives the identity:

   $$X_D[p_N(t) + t] \equiv X_S[p_N(t)]$$

Differentiating:

$$\frac{\mathrm{d}X_D}{\mathrm{d}p_G}\left(\frac{\mathrm{d}p_N}{\mathrm{d}t}+1\right)=\frac{\mathrm{d}X_S}{\mathrm{d}p_N}\left(\frac{\mathrm{d}p_N}{\mathrm{d}t}\right)$$

Solving gives:

$$\frac{\mathrm{d}p_N}{\mathrm{d}t}=\frac{\frac{\mathrm{d}X_D}{\mathrm{d}p_G}}{\frac{\mathrm{d}X_S}{\mathrm{d}p_N}-\frac{\mathrm{d}X_D}{\mathrm{d}p_G}}$$

and:

$$\begin{aligned}\frac{\mathrm{d}p_G}{\mathrm{d}t} &= \frac{\mathrm{d}p_N}{\mathrm{d}t}+1\\ &= \frac{\frac{\mathrm{d}X_S}{\mathrm{d}p_N}}{\frac{\mathrm{d}X_S}{\mathrm{d}p_N}-\frac{\mathrm{d}X_D}{\mathrm{d}p_G}}\end{aligned}$$

(c) We can actually say a little more. Given the standard assumptions:

$$\frac{\mathrm{d}X_D}{\mathrm{d}p_G}<0,\quad \frac{\mathrm{d}X_S}{\mathrm{d}p_N}>0$$

it follows that:

$$\frac{\mathrm{d}X_S}{\mathrm{d}p_N}-\frac{\mathrm{d}X_D}{\mathrm{d}p_G}>0>\frac{\mathrm{d}X_D}{\mathrm{d}p_G}$$

Therefore:

$$0>\frac{\mathrm{d}p_N}{\mathrm{d}t}>-1$$

$$1>\frac{\mathrm{d}p_G}{\mathrm{d}t}>0$$

The net price decreases with the tax and the gross price increases with the tax. In general neither change is dollar-for-dollar.

(d) If demand is flat ($\frac{\mathrm{d}X_D}{\mathrm{d}p_G}=0$), so inverse demand is vertical, or supply is vertical ($\frac{\mathrm{d}X_S}{\mathrm{d}p_N}=\infty$), so inverse supply is flat, then the tax is shifted entirely to the buyer ($\frac{\mathrm{d}p_N}{\mathrm{d}t}=0$, $\frac{\mathrm{d}p_G}{\mathrm{d}t}=1$).

4. Taxation and oligopoly

   (a) In the simplest versions of the model, there are a fixed number of identical firms, $n$, producing a single homogeneous product.

   The theory is easily extended to allow free entry (see Besley, "Commodity Taxation and Imperfect Competition: A Note on the Effects of Entry," *Journal of Public Economics* 40 (1989), 359-367).

   It has also been extended to allow for differentiated products.

   (b) Industry output:

$$Q_S \equiv \sum_{j=1}^{n} q_j$$

   Inverse demand:

$$p(Q_D)$$

   Cost function:

$$c(q_j)$$

   If we substitute aggregate supply into the inverse demand curve we obtain the market-clearing gross price for the good. Profits for firm $i$ are then:

$$\pi_i = p(Q_S)q_i - c(q_i) - tq_i \tag{1}$$

   We assume:

   i. $p'(Q_D) < 0$ at any $Q_D > 0$
   ii. $c'(q_j) > 0$ and $c(0) > 0$

   (c) With Cournot competition, firm $i$ maximizes (1) holding all other output levels constant.

   Taking the derivative with $q_i$:

$$\frac{d\pi_i}{dq_i} = p'(Q_S)q_i + p(Q_S) - c'(q_i) - t = 0 \tag{2}$$

   For firm $i$ the solution is $q_i(q_{-i}, t)$.

   We assume that a Cournot-Nash equilibrium exists, and we assume that in it all firms produce the same amount. Both assumptions are subsumed under the phrase, "assume a symmetric equilibrium exists."

   (d) Taxation, firm-level supply and aggregate supply.

   First, simplify the notation:

$$q_i(q_{-i}, t) \equiv q(t), \quad \text{all} \quad i$$

Substitute $q(t)$ back into (2), and use the fact that all firms produce the same amount of output in equilibrium:

$$p'[nq(t)]q(t) + p[nq(t)] - c'[q(t)] - t = 0 \tag{3}$$

Differentiate (3) with respect to $t$:

$$qp''n\frac{dq}{dt} + p'\frac{dq}{dt} + p'n\frac{dq}{dt} - c''\frac{dq}{dt} - 1 = 0$$

Therefore:

$$\begin{aligned} \frac{dq}{dt} &= [qp''n + np' + (p' - c'')]^{-1} \\ &= \frac{1}{p'}[(qp''n)/p' + n + (1 - (c''/p')]^{-1} \\ &= \frac{1}{p'}(E + n + k)^{-1} \end{aligned} \tag{4}$$

where:

$$E \equiv \frac{nqp''}{p'}$$

and

$$k \equiv 1 - \frac{c''}{p'}$$

Comments:

i. It may be confusing at first that all other firms' outputs are held fixed in (2) but they all change when we differentiate (3).
The difference in the thought experiments is as follows. In (2) we are solving for each firm's optimal output. In doing this, we suppose that each firm takes all other firms' outputs and the taxes as given. In equilibrium, each firm's output is optimal given the equilibrium level of output for all other firms.
Now suppose we change taxes. To find the new equilibrium we would repeat the previous optimization, solving for each firm's optimal output taking all other firms' outputs and the new taxes as given. It is critical to note, however, that other firms' output levels are in general at new values in the new equilibrium. The change in equilibrium supply by firm $i$ depends on the new tax rate and on the way in which supply by all other firms changes.
Equation (4) correctly captures both sets of changes. More precisely, if we recompute equilibrium supply by firm $i$ using (1) for a discrete change in $t$ and then consider how the ratio of the change in supply to the change in $t$ behaves as $t$ becomes small, then we obtain (4).

ii. The literature generally assumes:

$$E + n + k > 0 \tag{5}$$

It is necessary and sufficient for quantity to fall with taxes, and that is the most "reasonable" result.

At the moment, however, we know nothing about the signs of either $E$ or $k$. In fact, reasonable demand curves give $E < 0$, so (5) need not hold. For example, if $p(nq) = (nq)^\alpha$, with $\alpha < 0$, then:

$$\frac{\mathrm{d}p}{\mathrm{d}nq} = \alpha(nq)^{\alpha-1}$$

$$E = \frac{\mathrm{d}^2 p}{\mathrm{d}(nq)^2}\frac{nq}{p'} = \alpha(\alpha-1)(nq)^{\alpha-2}\frac{nq}{\alpha(nq)^{\alpha-1}} = \alpha - 1 < 0$$

On the other hand, constant returns to scale gives $c'' = 0$, in which case:

$$k = 1$$

iii. There are attempts to derive (5) using second order conditions for the optimization in (1) and "stability" conditions. These are not entirely successful.

Regarding second order conditions, differentiating (2) with $q_i$ and assuming $\pi_i$ is concave in $q_i$ gives:

$$\begin{aligned} \frac{d^2\pi_i}{dq_i^2} &= qp'' + p' + (p' - c'') \\ &= \frac{p'}{n}\left[\frac{nqp''}{p'} + n + n\left(1 - \frac{c''}{p'}\right)\right] \\ &= \frac{p'}{n}(E + n + nk) \\ &< 0 \end{aligned}$$

Therefore:

$$E + n + nk > 0 \tag{6}$$

Obviously (5) and (6) differ except for the case of monopoly ($n = 1$). Sometimes imposing some intuitive "stability" conditions can help. One standard stability condition is that the slope of the demand curve is smaller than the slope of the supply curve (above we assumed demand was downward sloping but we did not assume marginal cost was upward sloping, only that marginal cost was positive):

$$p' - c'' < 0$$

This with the fact $p' < 0$ implies:

$$k > 0 \tag{7}$$

Unfortunately, this gives us only that (5) implies (6), not the converse. Thus, even combining this stability condition with the second order condition does not imply that quantity falls with taxes.

Seade argues that an additional stability condition should be imposed (see Seade, "The Stability of Cournot Revisited," *Journal of Economic Theory* (1980), 15-27):

$$p''q + p' < -\frac{1}{n}[p' - c'']$$

If we multiply both sides by $n/p' < 0$ and rearrange then we do obtain (5). See that paper for details.

(e) We are interested in the effect of the tax on equilibrium gross price, net price, and profit for firm $i$. These follow from:

$$p_G(t) \equiv p[nq(t)] \tag{8}$$

$$p_N(t) \equiv p[nq(t)] - t \tag{9}$$

$$\pi(t) = p[nq(t)]q(t) - c[q(t)] - tq(t) \tag{10}$$

We assume that (5) holds.

(f) Taxation and equilibrium gross and net price

Differentiating (8) and using (4):

$$\begin{aligned} \frac{dp_G}{dt} &= p'n\frac{dq}{dt} \\ &= \left(\frac{n}{E+n+k}\right) \\ &> 0 \end{aligned}$$

The gross price increases with the tax.

A more interesting question is whether the tax can be overshifted, meaning this derivative exceeds 1. Since $\frac{dp_N}{dt} = \frac{dp_G}{dt} - 1$ this is equivalent to asking whether the net price increases or decreases with the tax.

Since the denominator above is positive, overshifting occurs if and only if:

$$n > E + n + k \tag{11}$$

or

$$E < -k$$

This can certainly occur. Using the example above with demand given by $p(nq) = (nq)^\alpha$ (with $\alpha < 0$) and production satisfying constant returns to scale, we have:

$$E = \alpha - 1 < -1 = -k$$

More generally, given just constant returns to scale, overshifting occurs if and only if $E < -1$ (Seade).

(g) Taxation and profits

Differentiating (10) and gathering terms gives:

$$\frac{d\pi}{dt} = -q\left(\frac{E+1+k}{E+n+k}\right)$$

Therefore

$$\frac{d\pi}{dt} > 0 \longleftrightarrow E+1+k < 0$$

i. This is not ruled out by the requirement $E+n+k>0$ unless $n=1$. So, profits must fall for monopoly, but they may increase for oligopoly.

ii. The intuition is that oligopolies produce more than the profit maximizing level of output. Additional taxes reduce industry output, so one expects industry revenues net of production costs to be higher in the new equilibrium (this is in Katz-Rosen). On the other hand, additional taxes are an additional cost. What actually happens to profits depends on how the growth in net revenue compares to the growth in costs from paying the tax. That is a subtle elasticity question answered by $E+1+k$.

iii. If profits increase then there must be overshifting:

$$\begin{aligned} \frac{d\pi}{dt} > 0 \quad &\longrightarrow \quad E+1+k < 0 \\ &\longrightarrow \quad E+1+k+(n-1) < n-1 \\ &\longrightarrow \quad E+n+k < n-1 \\ &\longrightarrow \quad E+n+k < n \\ &\longrightarrow \quad \frac{dp_G}{dt} > 1 \end{aligned}$$

iv. However, overshifting need not imply an increase in profits. Overshifting occurs if and only if $E+n+k<n$, which is consistent with both $E+n+k<0$ and $E+n+k>0$.