Lecture 11

Outline

1. Jorgenson and Yun
2. Slesnick
3. Feldstein

1. Jorgenson and Yun

   (a) Some further details of the model and the tax laws are in, "Tax Reform and US Economic Growth," *Journal of Political Economy*, Volume 98, Number 5, Part 2 (October 1990).

   However, it is not clear where a complete exposition of the model is available.

   (b) There is an infinitely-lived representative consumer with perfect foresight of all future prices and rates of return.

   In each period t, the consumer decides how much labor to supply (leisure to consume) and how much income to consume or save.

   The individual pays taxes that provide government services. The services enter the utility function in an additively separable manner: they are neither complements nor substitutes to other goods. Thus, government services have no impact on labor supply, consumption, or savings.

   (c) A single representative producer employs capital and labor services to produce outputs of consumption and investment goods.

   (d) Technology undergoes "Harrod-neutral" productivity growth at rate μ in each period.

      i. Consider isoquants with Labor on the horizontal axis and Capital on the vertical axis.
      Productivity growth is "Harrod-neutral" if, at any fixed level of output and K, each isoquant shifts to the left by a common percent.

      ii. More precisely, suppose technology changes from:

      $$Y = F(K, L)$$

      to:

      $$Y = F[K, (1+\mu)L]$$

      In effect the labor force is μ percent larger. In the standard terminology, population "in efficiency units" increases from L to $(1+\mu)L$.

This is not, however, the fraction by which the isoquants shift to the left. Given Y_0, K_0 and L_0, we need $\frac{L_0}{1+\mu}$ units of labor after the shift to produce Y_0. The percentage reduction in labor needed is

$$\frac{\frac{L_0}{1+\mu} - L_0}{L_0} = -\frac{\mu}{1+\mu}$$

This is the fraction by which the isoquants shift to the left.

iii. Note: if *both* Labor and Capital become more productive by the same proportion, then each isoquant shifts toward the origin.
This is called "Hicks-neutral" or "output-augmenting" growth.

So, in discrete time and with Harrod-neutral technical change at rate μ in each period, we have:

$$Y(t) = F[K(t), (1+\mu)^t L(t)]$$

Population at time t is $L(t)$, but population "in efficiency units" is $(1+\mu)^t L(t)$.

If we define C_t as aggregate full consumption, F_t as full consumption per-capita with population in efficiency units, and U_t as full consumption per-capita, we have:

$$F_t = \frac{C(t)}{(1+\mu)^t L(t)}$$

and

$$U_t = F_t(1+\mu)^t = \frac{C(t)}{L(t)}$$

(e) The inter-temporal welfare function, V, is a discounted sum of (what they call) atemporal welfare functions (U_t), taking into account population growth, intertemporal elasticity of substitution, and the rate of time preference.

(f) The representative consumer maximizes V subject to the intertemporal budget constraint. This equates full wealth, W, to the present value of full consumption over the whole future of the US economy. See equation (3).

(g) Alternatively, one can minimize full wealth subject to the time path of prices (PF), discount factors, (D), and a level of welfare (V). This gives the analog of the *expenditure function* for this model. Denote it:

$$W(PF, D, V)$$

The analog of the *equivalent variation* going from state 0 to state 1 is then:

$$\Delta W = W(PF_0, D_0, V_1) - W(PF_0, D_0, V_0)$$

(h) Definition of Excess Burden

They define the excess burden of taxation to be the equivalent variation of going from the post-tax state to the lump-sum tax state, where utility is higher.

While this is a perfectly well defined equivalent variation, it is not the standard one we have been using to define excess burden. The reference price vector here (the price vector in the initial state) is the post-tax price vector.

Intuitively, if $\mathcal{V}$ is the indirect intertemporal welfare function, the equivalent variation solves:

$$\mathcal{V}(PF_{86}, D_{86}, W_{86} + EV) = \mathcal{V}(PF_{L}, D_{L}, W_{L})$$

where W_{86} is full wealth with the 1986 tax structure and W_{L} is full wealth with the lump-sum tax structure.

We then have:

$$W_{86} + EV$$

$$= W[PF_{86}, D_{86}, \mathcal{V}(PF_{86}, D_{86}, W_{86} + EV)]$$

$$= W[PF_{86}, D_{86}, \mathcal{V}(PF_{L}, D_{L}, W_{L})]$$

Substituting $V_{L} = \mathcal{V}(PF_{L}, D_{L}, W_{L})$ and rearranging gives:

$$\begin{aligned} EV &= W(PF_{86}, D_{86}, V_{L}) - W_{86} \\ &= W(PF_{86}, D_{86}, V_{L}) - W(PF_{86}, D_{86}, V_{86}) \\ &= \Delta W \end{aligned}$$

Figure 1

(i) Last column of Table 2, row 9: AEC.

The "100%" means they compute the equivalent variation of going from the 1986 state to a state in which they reduce all tax rates by 100% (i.e., they eliminate all distorting taxes) and the lost revenue (which in this case is total tax revenue) is recovered through a lump sum tax. This figure is then divided by the lost revenue (again, total tax revenue in this case).

The other columns have an analogous interpretation. For example, "50%" means they compute the equivalent variation of going from the 1986 state to a state with all tax rates reduced 50% and the lost revenue recovered through a lump sum tax. This figure is then divided by the lost revenue.

The Average Efficiency Cost is .18.

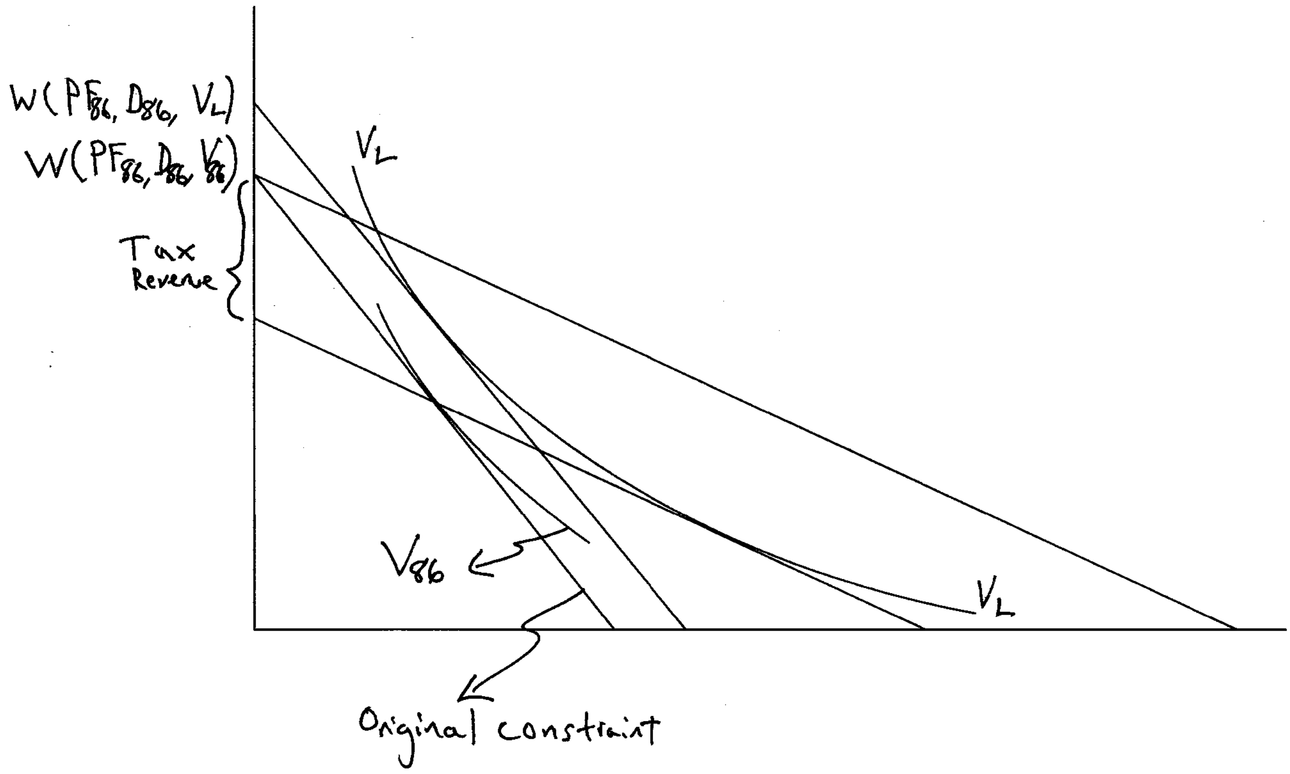
W(PF86, D86, VL)
W(PF86, D86, V86)
Tax Revenue
VL
V86
VL
Original constraint

Figure 1

(j) Last column of Table 2, row 9: MEC.

Formally, this is the equivalent variation of going from (1) a state in which all tax rates are 90% lower than in 1986, with the lost revenue recovered through a lump sum tax, to (2) a state in which all tax revenue is raised through a lump sum tax. This figure is then divided by the lost revenue.

This "marginal" calculation therefore corresponds roughly to adding the "first" distortion to an undistorted economy. One should expect it to be low since "the first distortion is free."

The Marginal Efficiency Cost is .040.

That *is* low!

(k) First column of Table 2, row 9: MEC and AEC.

View this as MEC. This is the equivalent variation of going from (1) the 1986 state to (2) a state in which all tax rates are 5% lower and the lost revenue is raised through a lump sum tax. This figure is then divided by the lost revenue.

One should expect this figure to be large since one is reducing tax rates in an economy with high distorting tax rates.

The Marginal Efficiency cost is .391.

(The AEC and MEC in the first column are always the same since they define the same set of changes. Think about it.)

2. Slesnick

(a) Slesnick develops measures of aggregate social welfare and aggregate deadweight loss that are consistent with each other.

(b) Starts with a social welfare function:

$$W(u, x)$$

where u is a vector of individual welfare *functions* and x is a matrix describing the consumption bundle of each individual. That is to say:

$$u = (W_1(x_1), ..., W_K(x_K))$$

where x_i is the ith column of x.

(c) The social expenditure function is the minimum aggregate expenditure needed to achieve a given *level* of social welfare, say W, at given prices. The expression in (2.2) is a little confusing. For given price vector p and value of social welfare W, they seem to want the solution (in the notation of the paper) to:

$$\underset{x_k \in \Re^N_+}{\text{Min}} \sum_{k=1}^{K} p x_k$$

$$\text{subject to:} \quad W[W_1(x_1), ..., W_K(x_K)] \geq W$$

This is $M(p, W)$.

(d) Aggregate deadweight loss.

We are given the economy with taxes and prices p^1. Let W^1 be the maximum level of welfare that can be achieved from any reallocation of the spending in this economy.

The aggregate deadweight loss is then:

$$L(p^1, p^0, W^1) = M(p^1, W^1) - M(p^0, W^1) - T(p^1, p^0, W^1)$$

$$T = \sum_{i=1}^{N} (p_i^1 - p_i^0) \sum_{k=1}^{K} x_{ik}^c (p^1, W_k^1)$$

THUS, aggregate deadweight loss is the excess over the compensated tax revenue that society would be willing to pay, with no loss in potential welfare, to shift from commodity taxation to lump sum taxation.

It is the (negative of the) equivalent variation of going from the equivalent (or "revenue-constant") lump-sum tax state to the commodity tax state.

(e) Average deadweight loss:

$$A(p^1, p^0, W^1) = \frac{L(p^1, p^0, W^1)}{T(p^1, p^0, W^1)}$$

(f) Commodity specific marginal deadweight loss:

$$MDL_i(p^1, p^0, W^1)$$

General marginal deadweight loss:

$$MDL(p^1, p^0, W^1)$$

(g) Monetary measure of social welfare

Slesnick then derives what he calls the "welfare theoretic counterpart" to this measure of deadweight loss.

This is the equivalent variation (using the social expenditure function) of going from the status quo welfare (W^0) to the welfare that would be achieved under lump-sum taxation (W^3):

$$C(p^1, W^0, W^3) = M(p^1, W^3) - M(p^1, W^0)$$

Note that while it should be possible to make everyone better off by moving to lump-sum taxation, the redistribution needed to achieve this is *not* part of the construction of W^3.

(This notation is also a bit confusing. If he had used W^* instead of W^1 above, he would have W^1 available here to denote the status quo level of social welfare.)

(h) Why this measure?

More "intuitive" measures of social welfare defined over vectors of individual spending require very strong assumptions to be consistent.

This is a technical and classical topic.

(i) His estimates of average deadweight loss ranges from .0387 (1981) to .0867 (1966).

These are much smaller than the figures in Jorgenson and Yun.

(j) "Aggregate deadweight loss is sometimes mistakenly interpreted as a measure of social welfare than an indicator of the loss of allocative efficiency. Therefore, it is of interest to compare the measures of aggregate excess burden with monetary measures of the change in social welfare that would result if the existing commodity tax structure is replaced with an equal yield lump sum tax."

Interestingly, in 8 of the 41 years, social welfare *decreases* moving from commodity taxation to lump sum taxation.

I think of the change in social welfare as capturing the "total" (efficiency plus equity) effect of the change. The shift to lump-sum taxation presents a potential Pareto improvement. Excess burden captures just the efficiency part.

Evidently, in some years, the actual equity loss more than offsets the efficiency gain.

3. Feldstein

(a) This paper builds on the "new tax responsiveness" literature.

The goal of that work is to estimate the elasticity of *taxable income* with respect to the *net of tax share* of income (i.e., with respect to $1 - t$).

Elsewhere, Feldstein estimated large elasticities. Here he wants to compute the excess burden implied by these elasticities.

(b) Feldstein works with a model in which consumer preferences can be written:

$$U(L, C, E, D)$$

where L is leisure, C is general consumption (i.e., using after-tax dollars), E is exclusions and D is deductions. The budget constraint is:

$$C = (1 - t)[w(1 - L) - E - D]$$

(c) The budget constraint can be rewritten as:

$$(1 + \tau)C = w(1 - L) - E - D$$

where:

$$1 + \tau \equiv \frac{1}{1-t}$$

"It is clear that the income tax is equivalent to an excise tax on ordinary consumption."

(d) The excess burden of this excise tax is now easily drawn (his Figure 1). Note that in this figure, the DD curve is the *compensated* demand curve. It is immediate that:

$$DWL = -.5\tau \mathrm{d}C$$

The goal is to write this in terms of the elasticity of compensated taxable income with respect to $1-t$.

(e) As a first step, Feldstein writes this in terms of the income-compensated elasticity of consumption. This elasticity is:

$$\epsilon_C = \frac{\mathrm{d}C}{\mathrm{d}(1+\tau)} \frac{1+\tau}{C}$$

Straightforward algebra gives:

$$DWL = -.5t^2(1-t)^{-1}\epsilon_C C$$

He then wants to show:

$$DWL = .5t^2(1-t)^{-1}\epsilon_T TI$$

where the income-compensated elasticity of TI is:

$$\epsilon_T = \frac{\mathrm{d}TI}{\mathrm{d}(1-t)} \frac{1-t}{TI}$$

(f) Feldstein's assertion is true if and only if:

$$-\epsilon_C C = \epsilon_T TI$$

This holds if and only if:

$$-\frac{\mathrm{d}C}{\mathrm{d}(1+\tau)} \frac{1+\tau}{C} C = \frac{\mathrm{d}TI}{\mathrm{d}(1-t)} \frac{1-t}{TI} TI$$

We have:

$$\frac{\mathrm{d}C}{\mathrm{d}(1-t)} = \frac{\mathrm{d}C}{\mathrm{d}(1+\tau)} \frac{\mathrm{d}(1+\tau)}{\mathrm{d}(1-t)} = -\frac{\mathrm{d}C}{\mathrm{d}(1+\tau)} \frac{1}{(1-t)^2}$$

Making the substitution gives:

$$(1-t)^2 \frac{\mathrm{d}C}{\mathrm{d}(1-t)} \frac{1+\tau}{C} C = \frac{\mathrm{d}TI}{\mathrm{d}(1-t)} \frac{1-t}{TI} TI$$

Clearing both sides, this is equivalent to:

$$\frac{dC}{d(1-t)} = \frac{dTI}{d(1-t)}$$

where both of these are derivatives of compensated demands.

(g) Feldstein does not give a formal argument for why this should be true. All he says is, "Since the uncompensated change in TI with respect to a tax change differs from the change in consumption only by the amount of the tax paid, the compensated effects are equal."

It is true that:

$$C = (1-t)TI$$

This, however, says that the *level* of TI differs from the *level* of consumption by the amount of the tax paid. This is not what Feldstein asserts.

If we consider comparative statics with respect to a change in $1-t$, we get:

$$\frac{dC}{d(1-t)} = TI + (1-t)\frac{dTI}{d(1-t)}$$

This is even further from what Feldstein asserts.

It is not clear whether the two compensated elasticities really are equal....

(h) Results

i. For the personal income tax (exclusive of social security) using $\epsilon_T = 1.04$, Feldstein establishes a 32% ratio of excess burden to tax revenue raised.

If the personal income tax is treated as an increment to the social security tax, the ratio becomes 52%.

ii. A 10% increase in all tax rates gives a ratio of extra excess burden to extra tax revenue of 78 cents, assuming no behavioral responses to reduce taxable income.

If one takes into account the reduction in taxable income, the ratio becomes $2.06.