Lecture 1

Outline

1. Gross and net prices
2. Unit tax
3. Ad valorem tax
4. General equilibrium: Consumer and producer prices
5. Budget sets with taxes: Net trades and taxation of net trades

1. Gross and net prices

   In this and the next two sections, think of an individual with income to spend on various goods. The only way to obtain a good to consume is to purchase it (you can not consume income directly). So, "quantity consumed" and "quantity purchased" mean the same thing.

   This will change later when we move to the general equilibrium setting and give consumers endowments of commodities.

   (a) Taxes, however defined, create a wedge between what the buyer pays and what the seller keeps for each unit of a good transacted.

   Rather than define taxes first, I will define the wedge and work back to the definitions of taxes.

   (b) p_G, or *gross price*: what the buyer pays per unit.

   p_N, or *net price*: what the seller keeps per unit.

   T, or *tax revenue per unit*: the revenue sent to (if positive) or received from (if negative) the government on each unit purchased.

   (c) It follows that:

   $$T = p_G - p_N \tag{1}$$

   Obviously:

   $$T > 0 \iff p_G > p_N$$

   (d) Our notation treats p_N, p_G and T as constants and they are not indexed by individual. This rules out the possibility that the tax revenue per unit might vary with the amount of the good purchased or the amount of other goods purchased or characteristics of the purchaser.

   Formally, the tax revenue is *proportional* to the number of units purchased, *separable* from all other purchases, and *anonymous*.

(e) Whether the buyer or seller sends the revenue to the government has no bearing on any economic issues. We do not discuss it further.

2. Unit tax

   (a) The simplest kind of tax is defined as the tax revenue per unit:

   $$t = p_G - p_N$$

   This is called a *unit tax*.

   (b) The definition implies that the revenue raised from a unit tax is proportional to the number of units purchased, separable from all other purchases and anonymous.

   These are basic properties of commodity taxes. As a practical matter, any tax that is paid at the point of sale must be very simple, although it need not be quite this simple. For example, if people are willing to show an identification card to the merchant (showing they are elderly or handicapped, for example) then this information could be used to impose a different tax rate.

   The income tax is not proportional, separable, nor anonymous, but it is also administered quite differently.

   We return to these themes later.

   (c) Equivalently, we could define a unit tax as an amount of money that a person by law sends to or receives from the government for each unit of good purchased.

3. Ad valorem tax

   (a) An ad valorem tax is a tax rate, specified as a fraction of either the gross or net price of a good, that a person by law sends to or receives from the government for each unit of good purchased.

   (b) If the tax is defined on the gross price, then the law essentially says, "tax revenue per unit must equal t% of what the buyer pays." Therefore:

   $$T = t_G p_G$$

   Using (1), this gives:

   $$p_N = (1 - t_G) p_G$$

   Consider an income tax that is a simple flat tax on the amount you are paid. This would be an ad valorem tax on the gross price of labor.

(c) If the tax is defined on the net price, then the law essentially says, "tax revenue per unit must equal t% of what the seller keeps." Therefore:

$$T = t_N p_N$$

Using (1), this gives:

$$p_G = (1 + t_N)p_N$$

I suspect that the sales tax works this way. The tax rate is defined as a markup over the prices posted in stores. This is surely the amount per unit that the seller sends to the government. The tax revenue per unit is therefore a fraction of what the seller keeps.

(d) Note that the two taxes raise the same revenue per unit as long as:

$$t_N p_N = t_G p_G$$

Therefore:

$$t_N = \frac{t_G}{1 - t_G}$$

Starting with the income tax above, the government could switch to a tax on the net price with no effect on any economic margins as long as it adjusted the rate according this formula.

4. General equilibrium: Consumer and producer prices

(a) For partial equilibrium tax analysis, the concepts of gross and net price are enough.

Even in this context, however, one should recall that the basic agents of a model are not buyers and sellers, they are consumers and producers. Gross and net prices are relevant to the behavior of both types of agents.

For example, a consumer buys goods at the gross price but sells labor at the net price. A producer sells goods at the net price but buys labor at the gross price:

	Buyer	Seller
Consumer	Goods (p_G)	Labor (p_N)
Producer	Labor (p_G)	Goods (p_N)

One must use the correct price in each agent's optimization problem. We will return to this when we examine tax incidence.

(b) For the formulation just described to be correct, "buyer" and "seller" must be exogenously determined. This is not true in general.

For example, suppose you have positive endowments of more than one commodity. You can then consume more than your endowment of one good, making you a buyer. Of course, you could also choose to consume less than your endowment, making you a seller. Which behavior you *choose* depends on the price of the good (and in general on the prices of all other goods).

"Buyer" and "seller" are endogenous.

Figure 1

(c) For this reason, the general equilibrium analysis of taxation takes *consumer prices* and *producer prices* as primitives.

The vector of consumer prices is generally denoted q. The vector of producer prices is generally denoted p. The difference is the tax vector:

$$t = q - p$$

(d) It is still true, however, that a tax raises money for the government if and only if the gross price exceeds the net price.

So, suppose $t_i > 0$. Necessarily then $q_i > p_i$. It follows that a *positive* tax rate raises revenue for the government if and only if the consumer price (q_i) is also the gross price and the producer price (p_i) is also the net price. The consumer must be buying and the producer selling. This would be true for anything the consumer wants in excess of what he has in his endowment vector. Most commodities would generally fall into this category.

The opposite holds for anything the consumer wants in a smaller amount than he has in his endowment vector. The consumer is then selling (keeping the net price) and the producer is buying (paying the gross price). If $q_i > p_i$ then the net price exceeds the gross price. A positive tax implies a unit *subsidy*.

Taxes on anything supplied by consumers must be *negative* for the government to receive revenue!

5. Budget sets with taxes: Net trades and taxation of net trades

(a) Most simple models of taxation are "income" models. Since you can not consume "income," there is no distinction between what you purchase and what you consume.

The optimal tax model uses a general equilibrium framework, and as such is an "endowment" model. This raises the question of whether the base for taxation is properly thought of as purchases or consumption.

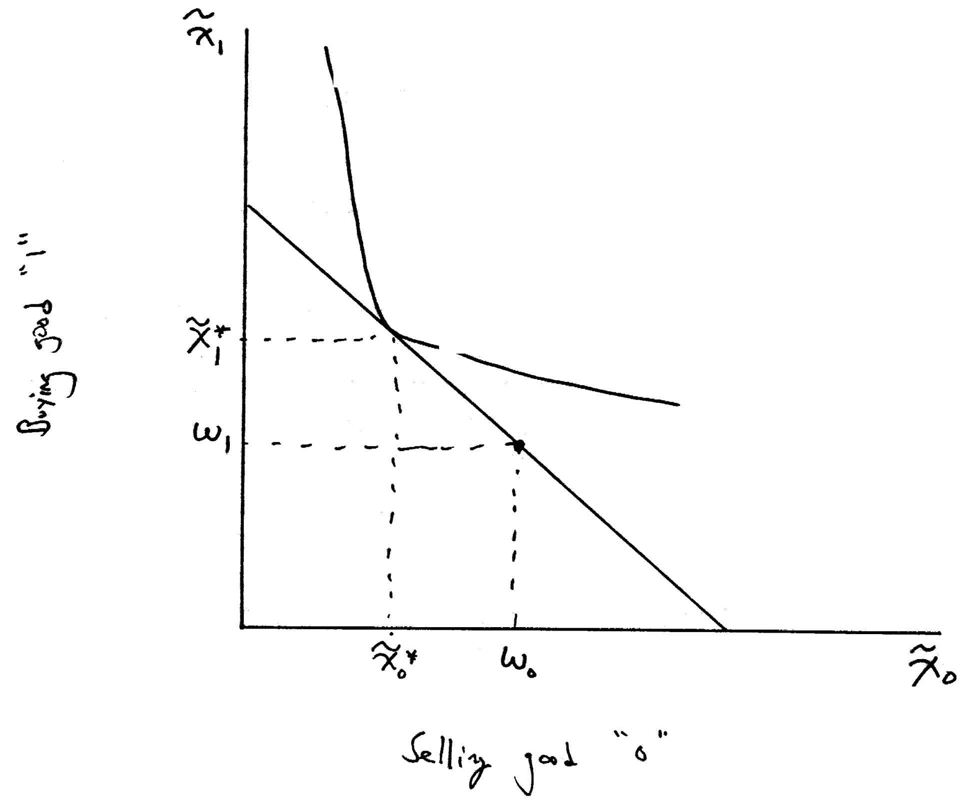

Figure 1

With endowments, whether you are a buyer or seller is endogenous. You are always a consumer.

(b) Recall the following concepts from the theory of the consumer (the vocabulary may be a bit different from what you've seen but the ideas should be familiar).

Consumption possibilities lie within a *consumption set*, $\tilde{X}$, and an element of this set is a *consumption vector*, $\tilde{x}$. Fix the notation:

$$\tilde{x} = (\tilde{x}_0, ..., \tilde{x}_n) \in \tilde{X}$$

One assumption always made is:

$$\tilde{X} \subset \Re_+^{n+1}$$

In general, however, other restrictions are meaningful. If $\tilde{x}_0$ is leisure time, then we always require this to be less than some number (like 24 hours) (we could also require it to be greater than some number, like 4 hours).

(c) We suppose this individual has preferences defined over $\tilde{X}$. He faces consumer prices q, has an endowment vector:

$$\omega \in \Re_+^{n+1}$$

and may also receive a payment of some amount of the numeraire good:

$$\pi \in \Re_+$$

The distinction between endowment and payment of numeraire good is irrelevant at the moment. It is important in a fully developed model in which the payment is endogenous (it depends on the equilibrium price vector). We include it now to make the notation here consistent with the notation later.

The budget set (in consumption space) for this consumer is:

$$\tilde{B}(q, \pi) = \{\tilde{x} \in \tilde{X} | q\tilde{x} \leq q\omega + \pi\}$$

(d) For example, suppose

$$\tilde{X} = \Re_+^2, \quad q = (1, 1), \quad \omega = (2, 3), \quad \pi = 0 \tag{2}$$

We have $q\omega = (1, 1)'(2, 3) = 5$, so the budget set is:

$$\tilde{x}_0 + \tilde{x}_1 \leq 5, \quad \tilde{x}_0 \geq 0, \ \tilde{x}_1 \geq 0$$

Figure 2A

Suppose we modify the consumption possibilities set:

$$\tilde{X} = \{\tilde{x} \in \Re_+^2 \mid \tilde{x}_0 \leq 1\}$$

Then the budget set is:

$$\tilde{x}_0 + \tilde{x}_1 \leq 5, \quad 1 \geq \tilde{x}_0 \geq 0, \ \tilde{x}_1 \geq 0$$

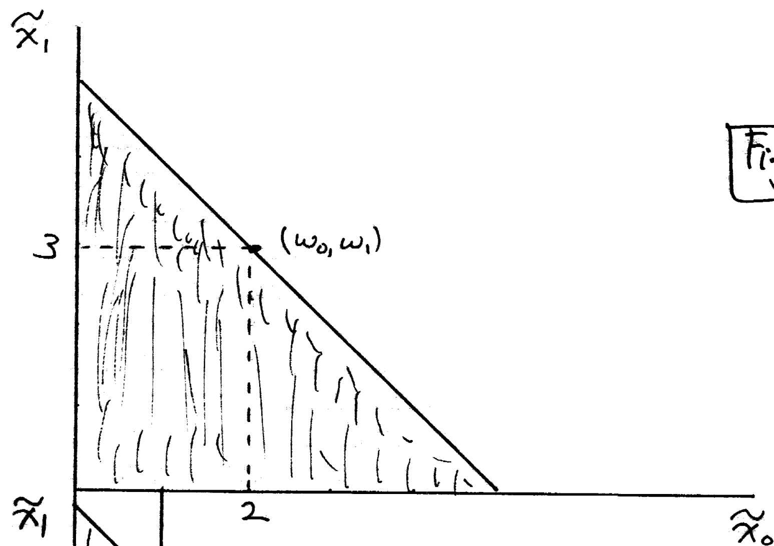

Figure 2A

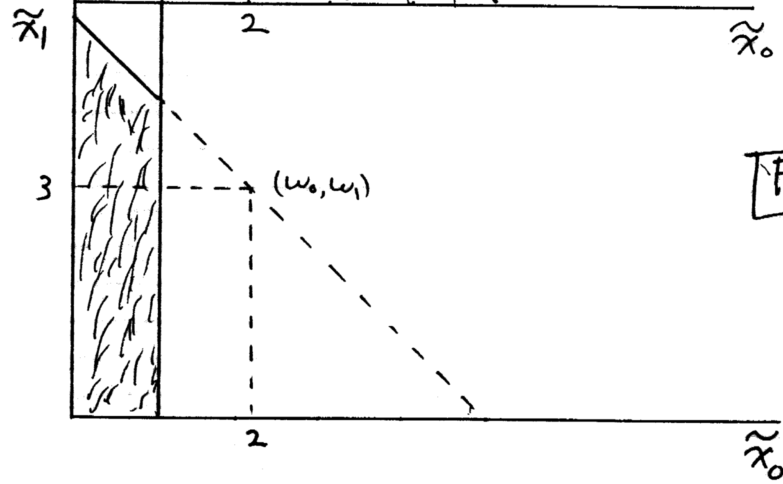

Figure 2B

Figure 2B

(e) Now suppose we put a tax on good "1" (the second good). This could mean a couple of things, since purchases and consumption are distinct.

(f) Suppose it means you pay the tax when you sell good "1" to buy good "0" *and* when you consume good "1" from your own endowment. We call this taxation of *gross consumption*.

Those are the only two things you can do with good "1" – sell it or consume it. The government takes some portion of each unit from you no matter what you do. That is the definition of an lump-sum tax.

If it can levy lump sum taxes then this is what it should do. An optimal tax model with taxation of gross consumption is not very interesting, at least with a single individual.

  i. *Note!* When there are many individuals, the question arises whether the government can vary the lump sum taxes (and transfers) across individuals.
     If it can, then again this is the solution and the model is not very interesting.
     If it can not, so it can use only *limited lump-sum taxation*, then the government can not redistribute resources with the lump-sum tax and there is room for commodity taxation. The government will have some incentive to tax goods consumed more heavily by those from whom it wants to take income, but will have to trade-off this goal against the efficiency effects.
     We return to this later.

(g) Suppose instead this means we put a tax on *net consumption* or *net trades*. We define net trades to be positive if you buy and negative if you sell:

$$x = \tilde{x} - \omega$$

If the tax base is net trades then individuals only pay the tax when they buy or sell.

Let's first derive the budget set in terms of net trades. The *net trades set*, X, is formally all $x \in \Re^{n+1}$ such that $x + \omega \in \tilde{X}$. The budget set (in net trades space) is:

$$B(q, \pi) = \{x \in X | qx \leq \pi\}$$

(we will not burden ourselves with extra notation).

(h) Recalling the data in (2), we have:

$$qx = (1, 1)'(x_0, x_1) = 0$$

The net trades set gives the restrictions:

$$x_i + \omega_i \geq 0, \quad i = 0, 1$$

The budget set is therefore:

$$x_0 + x_1 \leq 0, \quad x_0 \geq -2, \; x_1 \geq -3$$

Figure 3

If we add the requirement $\tilde{x}_0 \leq 1$, then in net trades space we require $x_0 + \omega_0 \leq 1$. The budget set is therefore:

$$x_0 + x_1 \leq 0, \quad -1 \geq x_0 \geq -2, \; x_1 \geq -3$$

(i) In order to study the budget set with taxes on net trades, suppose the consumer price rises by the full amount of any tax. Then we have a fixed vector q with t just added to it.[1] This gives:

$$B(q + t, \pi) = \{x \in X | (q + t)x \leq \pi\}$$

(j) Add the following tax vector to the data in (2):

$$t = (0, 1)$$

We then have:

$$(q + t)x = (1, 2)'(x_0, x_1) = 0$$

The budget set is therefore:

$$x_0 + 2x_1 \leq 0, \quad x_0 \geq -2, \; x_1 \geq -3$$

Figure 4

If we add the requirement $\tilde{x}_0 \leq 1$, then the budget set is:

$$x_0 + 2x_1 \leq 0, \quad -1 \geq x_0 \geq -2, \; x_1 \geq -3$$

(k) Note that there is nothing about taxing net trades that requires us to graph everything in net trades space. This is usually most convenient because of the way the picture links up to the netput vector of firms, but nothing forces this.

The budget constraint with taxes on net trades, graphed in consumption space, pivots at the endowment point (you can still consume your endowment vector).

[1] Equivalently, producer prices are fixed. This means $p(t) = p(0)$ for all t. But $p(0) = q(0)$, so $p(t) = q(0)$ for all t. We always have $q(t) = p(t) + t$, so making the substitution gives $q(t) = q(0) + t$.

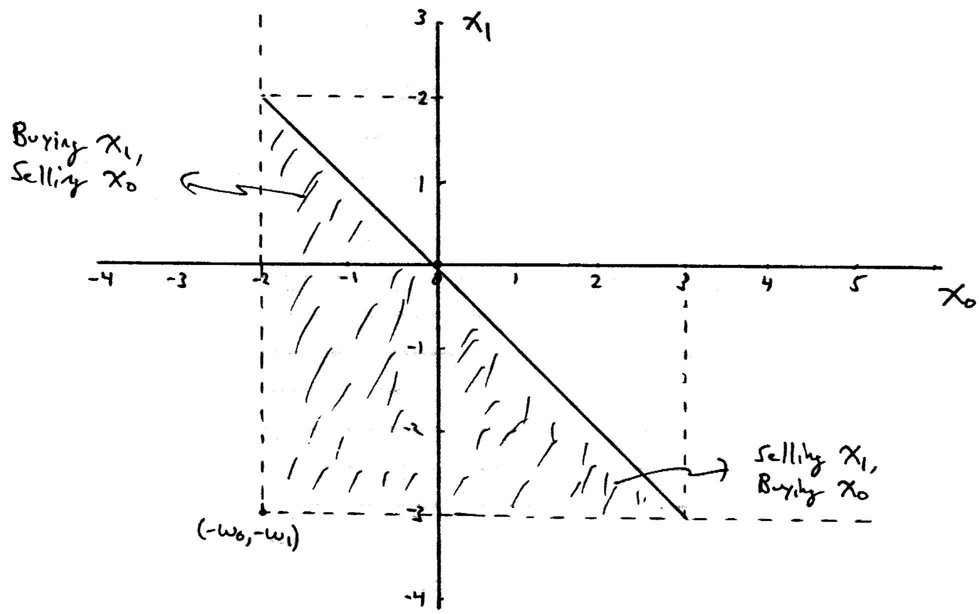

Figure 3

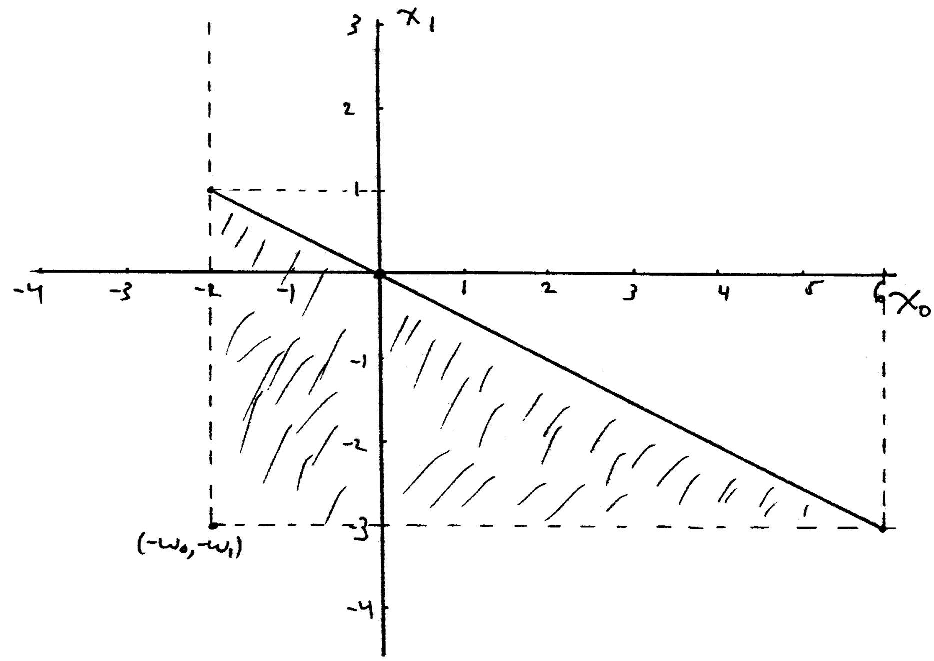

Figure 4